The cubit and shekel of the Torah

This is a long and detailed paper on the issue of the size of the cubit and the weight of the shekel. The paper is written in a condensed form with links to other papers (the majority of them in Hebrew) and presents a small fraction of a puzzle, which is called "The Truth of Land".

Our main claims are as follows:

a) The cubit of the Torah is exactly 48 cm. This is the cubit of the tabernacle and its vessels. This cubit defines also the units of volume: seah=3/40 cubit^3, kab=seah/6, lug=kab/4, cup or quarter=lug/4, egg=cup/1.5. The units of Torah are: eipha=3 seah, omer=eipha/10. We call it the cubit of man since it is the average length of the arm.

b) The cubit mentioned in Ezekiel 40:5 is exactly 51 cm. This is the cubit of the Temple, of the altar, of the molten sea of Solomon, of the inner court and its chambers, of the women's court, of the walls of Jerusalem in Nehemiah 3:13, of the Land of Israel and the future Jerusalem in Ezekiel 48. In particular, the area of the Land of Israel including the strip of Messiah is 1600x1600 sq. mils where mil=2000 cubits of 51 cm. The standard walking distance is 40 mils per day. This standard applies to all journeys in the Bible, in particular to the journeys of the Children of Israel in the desert. It is called in Chronicles II 3:3 "the original measure". It is also the cubit of Noah- the man of the land. We call this cubit- the cubit of land.

c) The shekel of Torah is exactly 17.28 gr. Its relation to the unit of volume is given by the equation: a cup of water weighs five shekels. When the children of Israel entered the land of Israel, the shekel was reduced to 2/3 of its original weight, i.e. to 11.52 gr. In the second Temple period it was increased by factor 6/5 to 13.824 gr. There was also a little bigger standard, by factor 25/24 or 14.4 gr. The last weight was increased by Gaonim and many Rishonim by mistake and by the ways of Providence by factor 6/5 and thus returned to its original value.

These claims are based on many evidences- material and literal. We will list them below.

1. The length of the cubit 48 cm is one of the two currently accepted values in the Torah communities. It is attributed to Rabbi Haim Naeh. The other value named the cubit of Hazon Yish, is 6/5 times greater, or 57.6 cm. The value of 48 cm fits the average length of the arm from the elbow till the end of the middle finger. The cubit relates to human measures as follows: it equals six hand widths, each hand width equals four digits, a digit being the width of the thumb measured in the widest place. The value of the last depends on how it is measured, by caliper applying pressure, or loosely as it is. In the first case, the width is 2 cm and the corresponding cubit is 48 cm. In the second case, the width is 2.4 cm and the corresponding cubit is 57.6 cm. The height of a man supposed to be 3 cubits. There are two interpretations to it: one is that the height is up to the shoulders, the second –up to the top of the head. The first

interpretation fits the smaller cubit, the second - the bigger one. Though the value of 48 cm is an approximate one, once it is accepted by the Halacha- the Jewish law, it becomes and absolute standard. We claim that the value of the French meter was set by the Providence to fit the Torah cubit so that the cubit would be expressed in integer number of centimeters. The measure of Hazon Yish is not a mistake. There is a controversy of two sages: Rabbi Meir and Rabbi Yehuda, regarding the size of the vessels in the tabernacle. According to Rabbi Meir the cubit of the vessels was six hand widths, while according to Rabbi Yehuda it was five hand widths. If the cubit of six hand widths of Rabbi Meir would be 48 cm and the one of Rabbi Yehuda would be 57.6 cm, then they would refer to the same reality of the vessels.

2. The cubit of Ezekiel can be calculated from the size of the Land of Israel. According to Ezekiel Ch. 48, the Land of Israel will be divided into 13 strips of equal width from north to south (Remark 1: the equal width of the strips is not stated explicitly in the text but is inferred by the Sages (Sifri, Ch. Haazinu, section 315) from the repeating word "one": Dan one, Asher one etc.). There will be seven strips of seven tribes to the north of Truma (donation) and five strips of five tribes to the south of Truma. The Truma itself will be a square of 25 by 25 thousand measuring reeds of 6 cubits, or 75 by 75 mils of 2000 cubits. This square will be split from north to south into the portion of the priests 30 mils wide, south to it the portion of Levies 30 mils wide, and south to it the portion of the servants of the city, 15 mils wide. To the east and to the west of Truma will be the domain of the Prince (Messiah). The Temple will be in the middle of the portion of the priests. Hence the difference between the latitude of the Temple and the latitude of the southern border of the Land of Israel will be 435 mils. Whatever is the size of the cubit, the future Land of Israel will cover the Sinai Peninsula. If the cubit is too large, the strip of the most southern tribe Gad will fall into the Red Sea. If it is too small, the southern part of Sinai will be no man's land. Thus, one should conclude that the southern border of the Land of Israel will coincide with the southern border of the Sinai Peninsula. Indeed, according to the commentary of Rashi on the verse (Ex. 23:31) "And I will set thy bounds from the Sea of Suf to the Sea of Pelishtim", the Red Sea will be the future southern border of Israel. The latitude of the center of the Dome of the Rock is 31.777719°. The latitude of the most southern point of Sinai is 27.7235°. This is the tip of the narrow protrusion of Ras Muhammad. The latitude of the southern shore of the mainland is 27.7806°. If we equate 435 mils to the distance between the latitude 31.777719° and 27.7235°, the cubit of Ezekiel is **51.65** cm. If we replace 27.7235° by 27.7806°, the cubit is **50.93** cm. The last value is very close to 51 cm we claimed in b). It seems unfair to Gad to include the narrow protrusion of Ras Muhammad in his 75 mils strip and thus exclude a significant area at his wide northern border. If we set the cubit of Ezekiel to be exactly 51 cm, the southern border of Gad passes at the latitude 27.775 deg, touching the corral reefs extending south from the mainland. (Remark 2: Yet Gad will apparently possess also the Head (Ras) of the protrusion thanks to the blessing of Moses in Devarim 33:20). Since Ezekiel uses the same cubit in measuring the altar, and since the measure of the altar will not be changed in the third Temple (Rambam, Beit Habhira, 2,3), and since the First and the Second Temple were measured by the same cubit as the altar, necessarily the cubit of the Temple was the cubit of Ezekiel.

The evidence we brought here about the size of the cubit of land is based on the accepted interpretation of the text in Ezekiel 48 (e.g. Rashi in Ez. 48:1 and Radak there) and can not be dismissed. One can yet argue whether this cubit is exactly 51 cm or approximately. In the following sections we will bring arguments that it is exactly 51 cm.

3. The most exact value of the cubit of land comes from the ladder of Yaakov. In the article about the sea of Solomon, we identified the strip of the Messiah around the world with the ladder of Yaakov. According to the Talmud (Chulin 91a) the ladder was 32000 mils wide. The circumference of the earth at the middle latitude of the strip of Messiah at the sea level fits this number of mils (the mil equals 2000 cubits of land) with relative accuracy of 1.6×10^{-4} (the circumference was computed by multiplying the diameter of the circle by 3 instead of π as explained in the article). This middle latitude crosses the mountains of Judea at their highest point which happened to be 1020 m or exactly 2000 cubits of land about the sea level. The circumference of the earth at this height fits the number 32000 with relative accuracy of **2×10^{-7}!**

According to the Sages (e.g. see Megila 3a) the area of Israel is 400 by 400 parsa, where parsa equals four mils. It turns out that the area of twelve tribes is 400 by 400 mils of land (relative accuracy of 2×10^{-4}). Since the width of the strip of Messiah is 75 mils from north to south, its area is 75x32000 mils. Together with 400x400 mils of the tribes it makes exactly 1600 by 1600 mils. The data about the borders of Israel could be found in the file. (It could be accessed by Google Earth and its contents could be read by Notepad. The coordinates of the borders of the tribes are stored in polygons of Gad, Zevulun etc. The area was computed on the ellipsoid of Clark at the sea level).

The file contains also the data about the journeys of the children of Israel. These journeys are reconstructed using the standard walking distance of 40 mils of land per day. Since certain key stops of the journeys are fixed, they define almost uniquely the corresponding cubit. The Hebrew reader is advised to read the article.

Using the cubit of land we have also recovered the walls of Nehemiah (see the article in Hebrew and the map in English). These walls fit the existing archeological data.

Another crucial piece of the puzzle which confirms the size of the cubit of land together with the cubit of man of 48 cm and the cubit of the world 52 cm, is the plan of the Holy Temple Mount of 500 by 500 cubits, the position of the Temple there and their relation to the existing expanded Temple Mount. This plan is explained in the article (in Hebrew) and shortly in the abstract (in English) and is shown in the map.

There are several existing monuments which are related to the plan in Ezekiel and to the size of the cubit of land. For example, the middle point of the eastern wall of the Tower of David lies exactly on the western border of the open space of 50 cubits mentioned in Ezekiel 45:2 and on the southern border of Yebusi (see the map in English and the plan or read the detailed article in Hebrew). The tomb of Rachel in Beit Lechem lays a "piece of land" (kivrat haaretz) north of Efrat (Genesis 32:16). We identified Efrat with the southern border of the future Jerusalem of 13.5 by 13.5 mils of land. The "piece of land" is the size of the field of Machpela bought by Abraham for 400 shekels. According to Gaon from Vilna, the area of the field

corresponding to this price was eight kur or 600,000 sq. cubits of 48 cm. Hence its size was 371.8 m. The top of the Dome of the tomb (which coincides with the center of the tombstone) is indeed 6.75 mils of 1.02 km less 371.8 m south of the center of Jerusalem. (Remark 3: the center of Jerusalem is 286 cubits east of the center of Holy of Holies along the axis of the Temple, at the eastern gate of the eastern court in Ezekiel 40:6. The axis of the First and Second Temples was directed about 6 deg north of east. The axis of the Third Temple will be directed exactly to the east. In this calculation we took the average between the past and future locations of the center). There is also an intricate connection between the Cave of Machpela and the center of Holy of Holies. It is explained in the Ch. 4 of the article (in Hebrew). The shrine above the cave is exactly 34 m or 100 feet of land wide (one foot equals 2/3 of the corresponding cubit). The length of the building is 116 cubits or 174 feet of land, such that the diagonal of it is as close as possible to the double width (Remark 4: 115 cubits would be a little better approximation, but this length can not be expressed in integer number of feet).

4.   One of direct archeological evidences of the cubit of land is the width of the double gate of the Temple Mount (see also the map). According to the Mishna Midot (Measures) Ch. 2,3 all gates of the Temple and Temple Mount where 10 cubits wide and 20 cubits tall. There are only two surviving gates: the Barcley gate in the western wall and the Double gate in the southern wall (the three arcs of the Triple gate are not the original ones). The Barcley gate was apparently the southern Parbar gate, mentioned by Josephus in Antiquities 15,410. This gate, however is not mentioned in the Mishna. (Remark 5: According to C.Warren (Plans, Elevations, Sections) the gate is 5.74 m or 10 cubits of Hazon Yish wide (according to the measurement of Eng. Meir Koznitz the interior width of the gate between the plastered walls is 5.55 m) and 9.79 m high (the height is not definite, it includes the sill course. Without this course, the height is 8.77 m)). The Double gate is one of the two Hulda gates mentioned in Midot 1,3. It served as the exit gate for Israel and as the entrance gate for mourners and banished. It has two openings, a part of the eastern one is seen from outside. In 1999 Prof. Doron Chen measured the width of the openings from inside (it is reported in Muqarnas, 18, p.9 and was confirmed to me privately). The eastern doorpost of the eastern opening is damaged but the western opening is intact. Its width is exactly 510 cm or 10 cubits of 51 cm. Yet the height of the openings is about 7.14 m or 14 cubits of 51 cm, not 20 cubits as supposed to be according to the Mishna (Remark 6: it is possible that the height of the gate was meant to be the diagonal of a square of 10 by 10 cubits with the standard approximation of √2 by 1.4 in the Talmud) . One should note that the Double gate is the southern entrance to the inner Hulda gate which was about 85 m north of it, at the southern border of the Holy Temple Mount (at the current exit in front of Al Aksa) (see the map). Thus, the statement of the Mishna does not necessarily apply to it. Yet it is reasonable to assume that the outer gate had the same width as the inner one since they where used by the same flow of people (the number of mourners and banished was negligible). The dimensions of the passage in south-north direction also fit the cubit of 51 cm (for more details see the article in Hebrew).

Another archeological evidence is the dimension of four huge stones in the western wall of the Temple Mount. These are the famous stones adjacent from south to the Warren gate seen in the Western Wall tunnel (Warren's gate is apparently the northern Parbar gate

mentioned by Josephus. However its northern doorpost and arched lintel are apparently not original)). The exact length of these stones is (from north to south): 504.0 cm, 1210.0 cm, 199.0 cm and 1350.0 cm (measured personally). There is a 1 cm gap between the first and the second stone. There are no gaps between other stones. The total length of four stones is 3264 cm (the number is exact up to few millimeters). This number equals 68 cubits of 48 cm or 64 cubits of 51 cm. Thus the total length of these stones preserves the size of the two basic cubits. The length of $64=2^6$ cubits is a very convenient one since, after measuring it with a rope, one can halve it successfully until reaching one cubit. There is a small door cut in the first stone (now sealed). Its dimensions are: width 80.0 cm and height 160.0 cm. These numbers are 10 and 20 palms of man of 8 cm. These are the standard proportions of the gates with the cubit replaced by a palm . The distance from the door till the northern edge of the stone is 112.0 cm. It is known that besides the standard cubit of 6 palms there was a bigger cubit of 7 palms and a smaller cubit of 5 palms. The length 112 cm is two cubits of 7 palms and the length of the stone 504 cm is 9 cubits of 7 palms. Nine cubits is also a standard unit of measure called a reed ("cane" in Hebrew). The height of the stone is 320 cm or 8 cubits of 5 palms. The importance of the first stone is that its northern edge serves as the southern doorpost of the Warren gate. If we connect this doorpost with the northern doorpost of Shoshan gate (the main eastern gate of the Temple Mount), this line (the red dashed line on the map) is perpendicular to the theoretical direction of the western wall (10.5522 deg west of north). Since the center of Shoshan gate was on the axis of symmetry of the Temple, then from it one can recover the location of the Temple. (Remark 7: Shoshan gate was destroyed but its location can be calculated from the Golden Gate. The theoretical distance between the center of Shoshan gate and the northern edge of the Golden Gate is 115+135/2 =365/2 cubits of land (see the map)). The actual direction of the segment of the wall formed by the four stones is 10.53 deg, more than reasonably close to the theoretical value of 10.5522 deg (see §15 in abstract). Apparently these huge stones were placed there to preserve the direction of the wall, the location of the doorpost of the Warren gate and the size of the cubits. (Remark 8: The sizes of other 3 stones may also have a meaning: 1350 cm=3x9x50 cm, 1210 cm=14x50 cm+10x51 cm. The smallest stone of 199 cm perhaps was meant to be 200 cm=4x50 cm, one centimeter was reduced to balance the 1 cm gap between the first and the second stones. The cubit of 50 cm is the average cubit of the Holy Temple Mount of 500 by 500 cubits. It is also one of the two world cubits and one of the two standard cubits exhibited on the Shoshan gate (see §6 below). Notice the total length of the three stones 3264-504=2760 cm can be represented in a unique was as a sum of cubits of 50 and 51 cm, namely 10 cubits of 51 plus 45 cubits of 50 cm. This sum was split into lengths of three stones. The largest one of three reeds of 9x50 cm or one tenth of a ris of 270 cubits, the smallest one of four cubits of 50 cm and the medium one of what is left. Recall that the length of four cubits is called the space occupied by a man and is a basic unit in Talmud). The accuracy of the cubit of 51 cm obtained from these stones is at least 3mm/64 or relative accuracy of 10^{-4}.

5.   The existence of a special cubit of the Temple, different from the cubit of Torah, is not mentioned explicitly in the Mishna or in the Talmud. Moreover, the Talmud in Menachot 97a calls the cubit of Ezekiel of a cubit and a palm – the cubit of six palms. Yet it is written in

Yiruvin 63a that the seah of Jerusalem was 6/5 times bigger than the seah of the desert (the one of the Torah). Rashi in Yoma 44b relates the introduction of this new seah to the entry into Jerusalem. Thus, it is implied that the new seah is related to the building of the First Temple. Now if we compare the volumes corresponding to the cubits of 51 and 48 cm, we obtain the ratio $(51/48)^3 = 1.19946\ldots$ or 6/5 with relative accuracy of 4.4×10^{-4}! It seems that the building of the Temple was accompanied by the increase of the cubit by the ratio of 17/16. Why the sages did not tell us that there was a Jerusalem cubit corresponding to the Jerusalem seah? Perhaps, because of the small discrepancy between the increase of the cubit and the increase of the seah.

The increase of the cubit in the Temple is hinted by the volume of the molten sea of Solomon. It is stated in Kings I,7,26 that the volume of the sea was 2000 bat or 6000 seah. The dimensions of the sea were: diameter 10 cubits and the height 5 cubits. The volume of a cylinder of such size with π approximated by 3 is 5000 seah. If, however, the cubit of the sea was the one of Jerusalem while the seah was of the desert, then the volume would be correct. In the article about the sea of Solomon, we have shown that, when taking into account the thickness of the walls of the sea and the two rows of balls on its upper brim, the volume is correct with the exact value of π. The relative accuracy of this result is 2×10^{-6}!

The fact that the cubit of the Temple was a special one is mentioned in the Midrash Rabba, Genesis, 31,10. Rabbi Yudan quotes the verse in Genesis 5:15 "And this is the fashion of which thou shalt make it (the arc) etc." and the verse in Chronicles II, 3 ,3 "the length by cubits after the ancient measure", and infers that the Temple was built by the same cubit as the Arc of Noah. This cubit had a distinct name, the "tevickin cubit". This name refers either to the arc or to some average measure. Indeed the cubit of 51 cm is an average of two cubits of the world of 50 and 52 cm (see §6 below). It is also a compromise between the cubit of Rabbi Haim Naeh of 48 cm and the cubit of Hazon Yish of 57.6 cm. Namely, instead of increasing the liner measure by ratio 6/5 it corresponds to the increase of the volume by this ratio. It is also the average between the two extreme values of the cubit: the cubit of Hazon Yish of 57.6 cm and the Roman cubit of 44.4 cm.

6. Besides the standard cubit of Moses of six palms, the Mishna (Keilim 17,9) says that there were two cubits on the tower of the Shoshan gate. One was exceeding the cubit of Moses by a half digit and the second one by a digit. If the cubit of Moses was 48 cm, then these two cubits were 49 and 50 cm. We saw that the cubit of 50 cm is present in the lengths of the three huge stones above. The cubit of 49 cm is the unit used in the design of the Tower of David (see the article in Hebrew). The horizontal dimension of the lower part of the tower (of 8 layers of stone) is 45 by 36 cubits of 49 cm . The middle part of the tower is 44 by 35 cubits of 49 cm (the upper part is from the middle ages).(Remark 9: We measured only the middle part. The north-west corner of the lower part is exposed and projects a cubit north and west relative to the middle part. The south-east corner does not project at all. The other corners are not exposed. Thus, we assumed that the theoretical dimensions of the lower part were by one cubit larger than of the middle part). The average perimeter is 160 cubits of 49 cm. This fits the description of the tower of Pazael by Josephus (Wars 5,4,3). According to him, the tower was 40 by 40

cubits. The crucial importance of the tower and likewise of the four huge stones in the Western Wall is that they define uniquely the location of the Temple.

Another cubit can be calculated from the size of the world of 24,000 mils mentioned in the Talmud (Pesahim 94). As it follows from the text there, this is the distance passed by the sun from dawn till the night. The last is equal 15 hours or 14.4 hours depending on two opinions. The distance passed by the sun along the tropic line is correspondingly 15/24x40075 km or 14.4/24x40075 km. If we equate this distance to 24,000 mils, the corresponding cubits are 52.18 cm or 50.09 cm. Since these cubits originate from the size of the world, we call them the cubits of world. By the reason explained in the next section we will round them to integer numbers of 50 and 52 cm.

7.   Apparently, the five cubits of 48, 49, 50, 51, 52 cm are hinted in the width of the court of the tabernacle. It is written in Exodus 27:13 that the court was 50 cubits wide. Yet it is not clear whether this number includes the one cubit width of the pillars. If it does, then the interior width of the court was 48 cubits. If it does not, than the exterior width was 52 cubits. According to the Braita (external Mishna) of the Art of the Tabernacle, the distance between the pillars was 4 cubits. Since there were 11 pillars along the width of court including the corners, the exterior width of the court was 51 cubits and the interior one was 49 cubits. Thus we obtain values ranging from 48 to 52 cubits corresponding to different opinions. Could all of them be called 50 cubits? Yes. The width of 48 is 50 cubits of 48 cm, the one of 49 is 50 cubits of 49 cm, etc. the one 52 is 50 cubits of 52 cm. Hence there is no contradiction between the interior and exterior dimensions. But how do we know that these cubits are measured by centimeters? If we wish to incorporate the two basic cubits of 48 and 51 cm in the sequence of measures which relate as numbers from 48 to 52, then there is no other possibility. Since the cubit of land is determined with relative accuracy of up to 2×10^{-7}, the rest of the cubits are determined with the same accuracy.

The controversy between different values of the cubit found its expression in the measures of the Temple Mount. According to Rashi in Chronicles I, 28:19, the size of the Temple Mount of 500 by 500 cubits is derived from the width of the court of the tabernacle and is hinted by the words "width fifty by fifty" in Exodus 27:18. The two extreme values of 48 and 52 cm became the units of the length of the south-east and north-west borders of the Temple Mount correspondingly. The value of 51 cm became the unit of length of the Temple and its courts and chambers. The cubits of 49 and 50 cm were exhibited on the Shoshan Gate. The double average cubit of 50 cm also became the standard international meter.(Remark 10: The palm of the Torah of 8 cm is also related to the "primitive inch" as inch=palm/π with relative accuracy of 2×10^{-5} . According to Charles Warren ("The ancient cubit and our weights and measures" p.25) the primitive inch=18/53 of the pyramid palm, p. palm=1/14 of double pyramid cubit, d.p. cubit=1/220 of the Great pyr. base, g.p. base=1/8 of one min. arc-length at the latitude 30 deg, one min. =1847.54 m. Notice that the diameter of the circle with the perimeter of the palm is a basic unit it the Jewish law).

8. The cubit of Torah of 48 cm defines uniquely the units of volume. As stated in several places in the Talmud (e.g. Yiruvin 4b), the size of a Mikve (a ritual pool of water) is one cubit by one cubit by three cubits high and contains 40 seah of water. Hence seah equals 8294.4 cm^3. The smaller measures are: hin=seah/2=4147.2 cm^3, kav=seah/6=1382.4 cm^3, lug=seah/24=345.6 cm^3, quarter (or standard cup)=lug/4=86.4 cm^3, egg=quarter/1.5=57.6 cm^3. (Remark 11: This is about the volume of average eggs (#3) in Israel today (weight 60-65 gr, volume 93% of the weight)) . The large measures are kur=homer=30 seah=248832 cm^3, bat=eiphah=kur/10 (Ezekiel 45)=24883.2 cm^3, omer=eiphah/10 =2488.32 cm^3.

9. The Torah units of volume are apparently related to Italian measures. The Mishna Keilim Ch. 17, 11 says: "the measures of liquid and dry are estimated by Italian- this is the desert (namely, Torah units)". Italian means the Roman, but it also may mean the Greek. Rome is several times called in the Talmud, "the Italy of Greece" (e.g. Shabat 56b, Megila 6b). Indeed Rome and Greece had a common culture. They also had common or related units of volume and weight. Roman basic large unit of volume was amphora or quadrantal which contained 80 Roman pounds of wine or water (e.g. see Smith dictionary). This weight was also equal to the Attic talent of 60 Attic minae of 100 drachmas. Twice of this volume was a basic Greek unit medimnus. The weight of the Roman pound is 324 gr. (Remark 12: we adopt the value given by Livio Stecchini in the Origin of Metrics §6 (his site was meanwhile removed). This value is equal exactly to 5000 English grain of 0.0648 grams of which 7000 make a pound avoirdupois of 453.60 grams). The corresponding Attic mina is 432 gr and the Attic tetradrachm 17.28 gr. These weights are confirmed by the weights of Greek coins. See the discussion below). Hence the quadrantal is 25920 cm^3. This volume is almost identical with the cube of the standard Roman foot of 29.6 cm (see the article). One third of the quadrantal is the Roman modius or Greek hecteis of 8640 cm^3.

This unit is exactly 25/24 of the seah of 8294.4 cm^3 while the quadrantal is 25/24 of the bat or eiphah. If we take the words of the Mishna in Keilim literally, then the seah of the Torah should be 8640 cm^3 and the corresponding cubit 48.658 cm. However, there is no archeological or geographical evidence to support such value. On the other hand, two measures that differ by factor 25/24 could be considered as two faces of the same unit, brutto and netto correspondingly (Remark 13: see §4 in the Origin of Metrics of Stecchini. He writes there in particular, that the cubes of the Egyptian foot of 30 cm and the cube of the Roman foot were related as 25:24. If Egyptian foot was 30 cm then the corresponding Egyptian small cubit of 45 cm, the cubit of the Torah and the cubit of land related exactly as 15:16:17). The fraction 1/24 of the volume of dough was the standard amount of chala (for a household; the amount for bakery was 1/48) and was given to the priests. The Talmud in Yiruvin 83a tells about modia condis measured by Rabbi to contain 217 eggs. The Talmud asks: if this modia was the seah of the desert it should have contained 144 eggs, if it was the seah of Jerusalem it should have contained 6/5 times more or about 173 eggs, if it was the seah of Zippori it should have contained 6/5 times more than the former or about 207 eggs. Then the Talmud suggests: perhaps this was the seah of Zippori that included chala of 1/24 (the total volume would be 216 eggs). Finally the Talmud concludes that this was the seah of Zippori but Rabbi was adding to each egg 1/20 of its volume (the total volume was supposed to be 217.7 but Rabbi counted

only whole eggs). We do not know what was the source of this modius but clearly, the Talmud suggests that it was 25/24 times larger than the standard one. The amount 1/24 was also added to the half shekel annual donations to the Temple and was considered as the price of exchange (so called "kalbon"). Another example is the golden dinarius. Its value according to the Talmud (Bechorot 50a) was 25 silver dinari. Tosfot, however, explains there that its weight was double the one of the dinarius of silver while the price of gold was twelve times the price of silver as in the time of David. Hence it was worth 24 silver dinari. The extra dinarius was the price of exchange.

All things considered, it seems that the Roman units of volume support the claim that the cubit of Torah was 48.00 cm. In the sequel we will bring evidence for this value also from Roman and Greek units of weight.

10. Now we turn to the discussion of the weight of the shekel and its relation to the units of volume. The main source of information about the shekel is the Talmud Bechorot 49b and 50a. The Mishna on p. 49b says "five selas of son (the redemption of the firstborn) by Tyrian maneh etc". The Talmud explains that Tyrian maneh is the maneh of the city of Tyre. The maneh (or mina) was equal 25 sela or tetra drachm. There are more than 300 Tyrian tetra drachms from years -285 till 60 on the site of ANS (American Numismatic Society). Their average weight is 14 gr. However, from the distribution of their weights we learn that the most frequent weight is in the range 14.2-14.25 grams (Remark 14: the histograms here and below include the weights corresponding to the left edge of the column but not to the right edge). When the coins are struck, their weights should distribute normally around the average. We see however that the distribution is uneven: it drops sharply to the right and decays slowly to the left. The reason is when a coin is damaged, its weight always declines, never increases. The most frequent weight should however survive. It is possible that because of the uniform wear, the whole graph is shifted a little left. Yet there is a surprising feature in the histogram: a pronounced local maximum at the range of 13.65-13.7 grams. The graph looks as a sum of two similar distributions, one (with majority of the coins) centered at 14.225 grams and the second (with minority) at 13.675 grams. The bump at the weight 13.9-13.95 looks as a result of summation of two tails of these distributions. The ratio of these two maxima is 14.225/13.675=1.04.., almost exactly 25/24. This is the ratio we mentioned in §9. It seems that there were two standards of the tetra drachm related as 25:24, the majority of the heavier kind. We will see below that the heavier standard was actually 14.4 gram, exactly 5/6 of the Attic tetra drachma. (Remark 15: Stecchini in the Origin of Metrics §3 and §6 explains that in many instances there was a discrepancy of 80/81 between basic weights. For example, Byzantine solidus was struck as 1/72 of a pound, which was 80/81 of the standard Roman pound of 324 gr. Notice that the value 14.225 is almost exactly 80/81 of 14.4 gr. May be this is the reason the Mishna refers to the Tyrian maneh and not to the actual Tyrian shekel). There are few nice coins with such weight. The corresponding maneh is 360 gram. The lighter standard thus is 24/25x360=**345.6 gram** or exactly **one lug** of water. The corresponding shekel is 13.824 gr.

The Talmud on p.49b mentions two other coins: Arabian dinar and "astira sarsia" (an old sela according to Rashi) by which one can arrive at the weight of the sela. Both are unknown to us. Then (at the beginning of p. 50a) Rabbi Yohanan says: "dinar hadraina traina

shaifa which is sold for 25 zuz, take off one sixths, the rest is for redemption of the son". After a correction, the Talmud concludes: "take off one zuz and one sixth (1+1/6x24=5), the rest (20) for redemption". Thus sela is 4 zuz, where each zuz is 1/25 of the said golden dinar. Rashi in Avoda Zara 52b explains that the words "hadraina traina" are names of the kings Traian and Hadrian, while word "shaifa" means "worn off". This description fits exactly the aureus struck by Hadrian. It had the legend IMP CAESAR TRAIAN HADRIANVS AVG. Its weight was much smaller than in times preceding Traian and Hadrian, so that one can say that it is "worn off". On the site of ANS there are 26 such coins. Their weights are 7.21 7.28 7.39 7.24 7.2 7.24 6.94 7.11 7.18 7.04 7.21 7.17 7.11 7.3 7.23 7.16 7.24 7.22 7.25 7.12 7.22 7.15 7.39 7.21 7.17 7.06 grams. Their average is 7.1960 and standard deviation quite small- 0.1. The distribution of the weights is symmetric around the average. This is an indication that the coins were not damaged. In general, golden coins are better preserved than the silver ones since they are nor circulating much. Thus, the above average can be trusted as the mean weight of the coin. We will round it to 7.2 grams. If the price of gold was 12 times the one of silver as implied by Tosfot in Bechorot 50a (Remark 16: This ratio is also mentioned in Roman and Greek sources, see Smith dictionary), then the value of this aureus was as 12x7.2=86.4 gr of silver. One zuz was thus 86.4/25=3.456 gr of silver and one sela 13.824 gr. This is the above light standard. But according to Tosfot the real value of the aureus was 24 zuz, extra zuz was the price of exchange. (Remark 17: According to Tosfot this is hinted in the words of Talmud, "take off one zuz and one sixth" instead of "take off one fifth"). Thus one zuz was 86.4/24=3.6 gr and shekel 14.4 gr- the heavier standard.

11. The weight of the shekel is also preserved by the coins struck in Judea during the Jewish war (years 66-70) and the revolt of Bar-Kohba (years 132-134). The latter however were struck over Roman coins. On the site there are several hundreds of shekels and half-shekels from the time of the Jewish war. We collected all published weights from there (see the file). Here is the histogram of the weights of the shekels and the histogram of the half-shekels. (Remark 18: The columns of the histogram are centered at points of one tenth of the gram, since several weights on the above site are given only with such accuracy). It is obvious that the most frequent weight of the shekels lies around 14.2 gr exactly as for the Tyrian tetra drachma. The distribution of the weights of the shekel is not symmetric, similar to the one of the tetra drachma. One can identify another local maximum at 13.8 gr. It is possible that the last maximum corresponds to the light standard of the shekel of 13.824 gr (Remark 19: Two weight standards of coins could coexist in the same mint in the same time and with the same die, if different minas were used to prepare the tokens). Now compare with the histogram for the half-shekels. The most frequent weight lays around 6.7 gr. Unlike the shekels, this global maximum is also the average weight of the coins. There are no additional standards of weight. Compare also with the histogram of Roman didrachma coined in Caesarea, Cappadocia during the reign of Traian (there are about 370 coins from this period on the site of ANS). Here again the most frequent weight is in the range 6.7-6.75 and is also the average weight. Since Roman pound at that time was divided into 96 denarii (see Smith) or 48 didrachma, the intended weight of the didrachma is 1/48 of the Roman pound of 324 gr, i.e. 6.75 gr. The collected data indicates that Jewish shekels were coined according to the Tyrian standard with, probably,

additional standard of 13.842 gr, while the half shekel was coined according to the standard of Roman didrachma. These half shekels were used in annual contribution to the Temple. According to the Mishna Shekalim 1, 7 one had to add to the half shekel a surcharge-"kalbon", 1/48 of the shekel. The total amount 25/24x6.75=7.03125 gr was more than half of the shekel of 13.842 gr but less than half of the heavy shekel of 14.4 gr or even of the average 14.2 gr. This is an indication that the Temple standard was the light one (Remark 20: According to Rabbi Meir in Shekalim 1, 7 the kalbon was 1/24 of the shekel. Hence the total amount was more than half of the heavy shekel. However, the Halacha-the legal ruling is not according to Rabbi Meir).

12. The above weight of the shekel is supported by the opinion of Rashi. In commentary to Exodus 21:32 Rashi wrote: "and the weight of the shekel is four goldens which is half ounce of the straight weight of Colonia". By Colonia is meant the city of Cologne (or Köln in German). It is agreed by the metrologists that the ounce of Cologne was about 29.2 grams (Remark 21: According to Stecchini in the Origin of Metrics §13 the Cologne ounce was identical with the ounce Tower of London. The latter was 450 grains or 29.16 grams). The corresponding shekel is about 14.6 gr, close to the heavier standard of Tyre of 14.4 gr. (Remark 22: This was also the opinion of the majority of the "Rishonim" in Ashkenaz-see the book of Rav Benish "Midot Veshiurey Torah, Ch. 22, comment 24). Yet Rashi contradicts himself. The goldens referred by him are the golden coins of Constantine as Rashi says explicitly in Avoda Zara 11a. There he identifies the maneh of Tyre with 25 sela, each sela is four dinar, and dinar weights as the golden of Constantine. These golden coins are the famous solidi which were coined continuously in the time period 500-1300. They had a very stable weight of about 4.44 grams (Remark 23: According to Stecchini in the Origin of Metrics §6 solidus was equal 1/72 of a reduced Roman pound of 80/81 of 324 grams, i.e. 4.444 grams). Rashi lived from 1040 till 1105. During his time till 1081 this golden coin was called Histamenon. Their average weight was around 4.37 gr (calculated from the data on the site of ANS). The corresponding sela of four such coins is almost exactly 6/5 of half ounce Cologne! After that, this coin was called Hyperperon. Their average weight was about the same as the Attic drachma, 4.32 gr. Hence the corresponding sela is the Attic tetra drachma of 17.28 gr. (Remark 24: How these two opinions of Rashi could coexist? Rabenu Gershom (960-1040) who preceded Rashi, writes in Bechorot 49b that dinar of silver is equal two and half pashut of the weight of iron. Pashut is the silver denier of Carolingian Empire. "The weight of iron" means the denier of weight (not the actual coin which was smaller) that was 1/240 of the pound of 12 Cologne ounces. Thus shekel of four dinars is equal half Cologne ounce-exactly as the second opinion of Rashi. Yet the weight of deniers decreased with time. At the time of Louis the Pious (813-840) the denier reached its pick of about 1.75 gr (this was apparently 1/240 of the Caroligian pound of 15 Cologne ounces less minimal surcharge of 1/25). The corresponding silver dinar of 2.5 pashut of this time was 4.375 gr (and without surcharge 4.556 gr), about the weight of the golden solidus. Rav Simcha bar Shmuel, a student of Rashi (died at 1105), wrote in Machzor Vitri that Tyrian maneh is the pound of Cologne which is hundred goldens of Constantine, where each golden weighs as two and half pashut of the weight of iron. It is possible that the original tradition of Jews in Europe linked the Tyrian shekel with four golden solidi and only

later linked it with ten deniers. As the deniers decreased, this weight became the half ounce of Cologne).

Rashi also relates the units of weight and volume. In Yiruvin 29a he says that the litra mentioned in the Talmud, is lug (Remark 25: This is also the opinion of Sheiltot (Achai Gaon) in chapter Beshalach, Sheilta 48. He identifies litra with rital of 12 ounces. Since ounce of Gaonim was 10 Babilonian drachmas, rital is 120 drachmas (see Midot Veshiurei Torah of Rabbi H.Benish, Ch. 13, 2)). In Bechorot 50a Rashi comments on the word litrin- maneh of 25 sela. Thus according to Rashi lug is equal to maneh of 25 sela. With the lighter standard of the sela of 13.824 gr one obtains the lug of 345.6 gr or 345.6 cm^3 of water, exactly the measure which corresponds to the cubit of Torah of 48 cm. With the heavier standard of 14.4 one obtains lug of 360 cm^3. If one identifies the sela with the half ounce of Cologne, then lug equals to 364.5 cm^3. Finally, if the sela is equated with the weight of four goldens of Constantine in the last 24 years of Rashi, then the lug according to Rashi is about the Attic mina of 432 gr. This lug is 25/24 times larger than the Jerusalem lug according to our claim. We think that it is legitimate to disregard the small difference between two standards of Rashi and the corresponding standards of Tyrian and Attic tetra drachma (Remark 26: Due to fluctuations and changes in the weights of the coins, it is hard to expect that the standard weight of the shekel could be traced in the time of Rashi with absolute precision). Thus, up to a known factor of 25/24, the measure of lug implied by Rashi coincides with the ones which correspond to the two basic cubits: 48 and 51 cm.

13. The relation between the units of volume and weight could be inferred from the Mishna, Trumot 10, 8: "Impure fish which was pickled with pure fish, each cask that contains double seah, if it contains the weight of 10 zuz of Yehuda which are five sela of Galil of impure fish, its brine is forbidden". The Yerushalmi Talmud, Trumot 53a comments on the Mishna: "How much is seah- twenty four lug, how much is lug –two litra, how much is litra- hundred zin. Thus each zin is one of nine hundred and sixty". All commentators of this Mishna understand from the language of Yerushalmi that the ratio of the weight of the impure fish to the total weight is 1:960. Hence 2 seah weighs 960 times ten zuz of Yehuda and one lug weighs 48 times less- 200 zuz. The question is: what are the zuz of Yehuda? Mishna in several places (e.g. Ketubot 5, 9) mentions the sela of Yehuda that was twice the sela of Galil. Hence zuz of Yehuda is a quarter of sela. Indeed, quarter of sela in Talmud is always called dinar or zuz. The standard sela in the Second Temple period is the Tyrian tetra-drachm (see § 10). If the lug of brine (or water) weighs 200 dinar or 50 sela, then its volume is 691.2 cm^3 (according to the light standard of 13.824 gr) or 720 cm^3 (according to the heavy standard of 14.4 gr). If this is the volume of the lug of Torah, then the corresponding cubit is about 60.5 cm or 61.3 cm. By all means these are impossible measures.

One possibility to explain the Mishna is to say that the measures of volume there are not of Torah but of Jerusalem that were 6/5 of the former, or of Zippori that were 6/5 of the latter. (Remark 27: the last was suggested by Rav Kanevsky in his publication "The measures of the Torah", though he used an exaggerated weight of the dinar of 4.80 gr). If the lug of Jerusalem was 720 cm^3, then the lug of the Torah was 5/6 times less - 600 cm^3. The corresponding cubit is 57.69 cm, almost exactly the measure of Hazon Yish of 57.6 cm! If the sela of Yehuda was

the Attic tetra drachma of 17.28 gr and the measures of volume are of Zippori, then the weight of 50 sela and the volume of the lug are increased by the same factor 6/5. Hence we obtain again the same cubit of 57.69 cm. According to Yerushalmi litra is equal to hundred zuz of Yehuda or 25 sela. In the former case litra is the Tyrian maneh of 360 gr and the latter case – the Attic maneh of 432 gr.

Another possibility is to say that the measures of volume in the Mishna are of the Torah but the sela of Yehuda was not the usual Tyrian sela. What was it? We suggest that it was not a coin (since there where no Jewish silver coins during the Second Temple history but only during four years of the Jewish war) but a standard weight of half sela or beka which every Jew was contributing annually to the Temple. This weight was half of the light standard of 13.824 gr, i.e. 6.912 gr. We saw in § 11 that the weight of didrachma was 6.75 gr. Jews were adding to didrachma a surcharge –kalbon of 1/48 sela, in particular to cover the difference between two weights. Since this weight of 6.912 gr was a unique Jewish standard, it had a special name- the sela of Yehuda. Notice that half sela is called by Mishna (e.g. Baba Matzia 34a) a shekel, and also the usual shekel is called sela. Thus the weight of half sela could be called sela. (Remark 28: The word sela in Hebrew also means a rock. It is possible that with regard to money, this name means a basic fixed weight). The zuzim of Yehuda were also not coins but weights, quarter of the sela of Yehuda, likewise the usual zuz was the quarter of the usual sela. Yerushalmi calls these zuzim- zinim and one hundred zinim – a litra, since one hundred of usual zuzim is called litra. The sela of Galil was a weight of 6.912/2=3.456 gr. It was a little heavier than the Roman dinarius of 324/96=3.375 gr. Therefore it had a special name.

Why there were different units of weight in Yehuda and Galil? The Mishna Nedarim 2, 4 says that if a person from Yehuda makes a vow that is related to truma (contribution), he means by it the contribution of the half shekels in the Temple, but if a person is from Galil he means the contribution to the kohanim. The reason is that the people from Yehuda were close to the Temple and hence more related to it. Since the half shekel was the main weight in the Temple, it was the standard weight in Yehuda. In Galil the standard weight was a quarter of the shekel since it was close to the dinar- the main coin in these days.

If sela of Yehuda is not the standard sela, one can reconcile the opinion of Rashi with the above Yerushalmi. Recall that according to Rashi litra is the weight of the lug (of water) and equals to 25 usual sela. But according to Yerushalmi litra is a half of lug and equals 25 sela of Yehuda. Conclusion: sela of Yehuda is a half of a usual sela and the corresponding litra is a half of the usual litra.

14. Now we will discuss the opinion of the Gaoinim about the weight of the shekel and the relation between the units of weight and volume. Apparently, all Gaonim agree that the shekel of the Talmud weighed four golden Islamic dinars and the shekel the Torah was 5/6 of it. This was also the opinion of Rif (Rav Yitchak Elfasi) and of Rambam. Since the weight of the golden dinar (so called mitqal) was about 4.25 gr, the corresponding shekel is about 17 gr, close the Attic tetra drachm of 17.28 gr. The shekel of the Torah is thus 14.17 gr, almost the same as the Tyrian shekel and the Israel shekel of the Jewish war.

As for the relation between the weight and the volume there are three opinions:

a) lug is equal 100 dirham

b) lug is equal rital (or ratl) of 120 dirham

c) lug is equal to 100 dinar.

Recall that dirham was a basic Islamic silver coin. Its official weight as established by Umar Ibn al-Khattab was 0.7 of the dinar, i.e. about 2.97 gr. There was also a lighter standard of 2/3 of the dinar (Kaftor Vaperach, also modern scholars W. Hintz, etc). The first opinion is of Rav Hilai Gaon and of others (see Rav Benish, Ch.13, 1). This is also the opinion of Rif (Remark 29: This is the version printed in the pages of Rif on Yiruvin 82b. However the first version of Rif was as the third opinion). This is apparently the opinion of Rambam in Mishne Torah, Yiruvin Ch. 1, 12. There he writes that quarter of lug of water of wine weighs 17.5 dinar. Hence lug is 70 dinar or 100 dirham according to the official ratio between them. (Remark 30: In his commentary on the Mishna, Rambam writes that the quarter of the lug of wine weighs 26 dirham and of water 27 dirham. If one assumes that Rambam was weighing with the light dirhams, there no contradiction between this weight and the weight in Mishne Torah). The second opinion is of Sheiltot (Rav Achai Gaon) and essentially of Rashi since 120 dirham weigh as 100 Tyrian drachmas. The third opinion is of Rabenu Hananel and others (see Rav Benish, Ch. 13, 3). The second opinion corresponds to the cubit of 48 cm, the third to the cubit of 51 cm, the first to the cubit of about 45.66 cm. (Remark 31: The last cubit is close to the English standard. The English foot is 30.48 cm and the corresponding cubit is 1.5 ft or 45.72 cm).

Apparently these three opinions are based on the above Yerushalmi Trumot. According to Yerushalmi lug is equal 100 sela of Galil. Those who followed the first opinion thought that the sela of Galil was the same as dirham. According to the second opinion sela of Galil was 1.2 of dirham or about the same as 5/6 of mitqal. The latter was supposed to be the weight of the dinar of the Torah. Hence according to the second opinion the sela of Galil was the dinar of Torah. According to the third opinion the sela of Galil was the mitqal or the standard dinar of the Talmud. We will see in the sequel that dirham was essentially the weight of the dinar of the first Temple. (Remark 32: It is possible that Jews, who lived in Babylon continuously since the first exile, kept the old standard and identified it with the Islamic dirham). Thus according to all opinions the sela of Galil was a quarter of normative sela and the sela of Yehuda was half of the normative sela (Remark 33: To justify the cubit of Hazon Yish, one can say that the weight of lug expressed by some opinions was not of water but of the flour or bread or dough. Since the specific weight of flour according to Rambam is 2/3 of water (this is apparently the free falling wheat flour), the weight of lug of water according to the third opinion results in a cubit of 58.87 and according to the second opinion – about 55.5 cm. One should also say that there is no connection between the above Yerushalmi and the opinions of Gaonim).

15. To summarize the discussion in §§10-14, there are two basic opinions about the weight of the shekel:

a) The Tyrian tetra drachm or the Israeli shekel of the Jewish war or half of the ounce of Cologne;

b) The Attic tetra drachm or four golden Islamic dinari or four Byzantine solidi at the and of 11-th century.

The ratio between these two categories of weights is about 5:6 or even 4:5 (between the light standard of the shekel of 13.824 and the Attic tetra drachm of 17.28 gr). On the other hand the Talmud (Bechorot 50a) says that the weight of the shekel was increased by the factor 6/5. Therefore one is tempted to propose that opinion a) applies to the shekel of the Torah while the opinion b) to the increased shekel. We will show that this is not a valid proposition.

First let us quote the Talmud there. We brought in §10 the Mishna and the discussion in the Talmud about the five sela of redemption of the firstborn ending with the words of Rabbi Yohanan. According to him this amount is equal 20 silver dinars where 25 of them are paid for the golden dinar of Hadrian. We have shown that both the Tyrian sela of 14.4 gr and the golden dinar (or aureus) of Hadrian of 7.2 gr give the same weight of the sela. The next sentence in the Talmud says that these 20 weights of dinar are equal to 28 zuz and a half and a half of danka (one sixth). Rashi explains there that this number is equal to 20/0.7 of Arabian dinars (Remark 34: Instead of a half of one sixth one should have said "a half of one seventh". Apparently, this text used the standard unit danka instead of 1/7 as an approximation. It is also possible that the ratio 1/6 was taken from inside. Namely, after you take away 1/7 from a zuz, it becomes 1/6 of what is left). Rashi says at the end that this sentence is apparently an addition to the Talmud since he found it in the answers of Gaonim. It is known that the Arabic dirham was indeed 7/10 of the golden mitqal and that Gaonim thought that this mitqal was equal to the weight of the dinar of the Talmud. However in reality the mitqal was 6/5 times heavier than the dinar of Talmud.

The following sentence in the Talmud says: "Said Raba: the sela of Torah is three and one third (dinar) since it is written "twenty gerah is shekel"(Exodus 30:13) and translated (by Unculus) "twenty meah" and a Baraita (external Mishna) says "six meah is a dinar". Thus according to Raba the shekel of Torah was 5/6 of the shekel of four dinar or 24 meah. The Talmud obviously refers to the shekel of the Mishna and the one of Rabbi Yohanan as the increased one. Thus the original shekel of the Torah was 5/6 of 14.4 gr (the heavy standard) or 5/6 of 13.824 gr (the light standard). The corresponding weights are 12 and 11.52 gr.

16. The above weights of the shekel of the First Temple period are confirmed by stone weights of the Kingdom of Yehuda. (Remark 35: There are no coins found in Israel or elsewhere prior to the Babylonian exile. It is believed by scholars that there were no coins but weights or ring money. Thus whenever Torah mentions shekels it means silver of a given weight). In the book of Raz Kletter, Economic Keystones: The Weight System of the Kingdom of Judeah (JSOT Supplement 276), Sheffield, 1998, are listed several hundreds of these weights. These are the weights of shekel and its multiples, the weights of beka, nesef, pym and of multiples of gerah (see the file) .The weights of beka, nesef and pym have an inscribed name on them. The weights of gerah do not have a name but a number of units. They are identified by their small weight. The weights of shekel are marked by a specific symbol and the number of units. (Remark 36: It is believed that this is a symbol of Royalty. In my opinion it resembles the form of the strip of Tefilin of the head. The king was anointed by smearing oil around his

head). Since its weight is very close to two beka, it is believed that this is the shekel. The name of beka appears in Torah and means half shekel. The names nesef and pym are not mentioned in Tanach or Talmud.

Now we present the statistical data about these weights. The weights of shekels and their multiples are shown on the histogram (Remark 37: There are 39 shekels, 38 two shekels, 53 four shekels, 55 eight shekels, 1 twelve shekel, 3 sixteen shekels, 3 twenty shekels and 2 forty shekels. Each multiple weight is replaced by one weight of the corresponding shekel). The distribution is quite compact and symmetric. The most frequent weight lies in the range 11.38-11.40 gr. According to our calculation the total average is 11.31 (of 194 weights), standard deviation 0.45. If one removes all weights that deviate from the average by more than the standard deviation, the new average is 11.339 (of 169 weights) and the new standard deviation is 0.21. The standard deviation of the average is 0.016. (Remark 38: This holds under the assumption that all stone weights have the same distribution).

There are 33 beka weights. Their distribution is not symmetric. The most frequent weight lies in the range 5.8-5.85. The average weight is 6.03, standard deviation 0.37. After removal of exceptional weights as above, the average becomes 5.96 gr (of 22 weights), standard deviation 0.18, and the standard deviation of the average 0.04. Thus the average weight of two beka 11.92 significantly exceeds the average weight 11.34 of the shekel. The most frequent weight of two beka 11.65 is also significantly larger than 11.39 of the shekel.

Now observe the distribution of the gerah weights (all multiple weights were replaced by the corresponding weight of one gerah). The most frequent weight is in the range 0.53-0.54 gr, the average 0.554 (of 83 weights), standard deviation 0.063- relatively very large. After removal of exceptional weights as above, the new average is 0.552 (out of 59 weights), standard deviation 0.03, standard deviation of the average is 0.004. Thus this average is quite stable and almost coincides with the most frequent weight. The surprising fact is that this weight of about 0.54-0.55 gr does not fit the weight of the shekel or the weight of two beka! Indeed, according to Torah (Exodus 30:13) the shekel is 20 gerah. The corresponding weight 10.8-11 gr is significantly smaller than 11.34. If the shekel was 24 gerah, the corresponding weight is 12.96-13.2, significantly larger than 11.92 gr.

We will now proceed with the weights of nesef and pym. There are

57 weights of nesef and 47 of pym. Here is the histogram of nesef. The most frequent weights are in the range 10-10.1 and 9.6-9.7. The average is 9.59, standard deviation 0.53. Without exceptional weights, average 9.68 (of 49 weights), standard deviation 0.27, standard deviation of the average 0.04. The scholars suggested that nesef is 5/6 of the shekel. The value 6/5x9.68=11.616 fits the most frequent weight of beka of 5.8-5.85 gr. Here is the histogram of pym. The most frequent weights are in the range 7.8-8 gr. The average is 7.782, standard deviation 0.29. Without exceptional weights, the average is 7.7796, standard deviation 0.14, standard deviation of the average 0.025. We see that the average is stable and is also close to the range of the most frequent weights. The scholars suggested that pym is 2/3 of the shekel. The value 3/2x7.78=11.67 fits nicely the most frequent weight of beka. The average weight 9.68 of nesef is close to 18 times the weight of gerah 18x0.54=9.72. The weight of pym, however, is not close to a multiple of gerah.

Notice that the expected value of the shekel 11.52 (the light standard) as deduced at the end of §15, lies in the middle between the average weight 11.34 of the shekel stone weights and the most frequent double beka weight of 11.6-11.7 gr. The heavy standard of 12 gr fits the average weight of the double beka of 11.92 gr. Thus the stone weights from the period of the First Temple support the expected values.

Yet there remains the severe problem of inconsistency between the weight of gerah and the weight of shekel of any standard. In order to solve it we will translate the calculated averages into units of pruta. Recall that according to our original claim the weight of the shekel of Torah was 17.28 gr. It was divided into 960 prutot of 0.018 gr or 20 gerah of 48 pruta. When children of Israel entered the Land, the shekel was reduced to 640 prutot of the above weight. The average weight of gerah of 0.552 gr is 30.7 prutot and the most frequent weight 0.54-0.55 gr is 30-30.5 prutot. Since we expect all weights to be multiples of pruta, the most appropriate approximation is gera=30 pruta=0.54 gr. This number divides 960/30=32. The average weight of the shekel stones 11.339 gr is almost exactly 630 pruta or 21 gera. The average weight of nesef is 9.68 gr or 537.8 pruta. It is very close to 18 gera=540 pruta=9.72 gr. This weight is in the range of one standard deviation of the average 0.04 (see above). Hence nesef is not 5/6 of the shekel but 6/7 of it. Indeed, if shekel is not equal to 24 gera, there is no reason for the weight of 5/6 of it. The average weight of pym is very stable, 7.78 gr. It is equal 432.2 pruta. We round it to 432 pruta or 7.776 gr. The number 432 is very "nice": $432=2^4 \times 3^3$ but it is not 2/3 of the shekel 630 pruta or of the theoretical shekel of 640 pruta. The average weight of beka 5.96 gr is 331.1 pruta. It is very close to 330 pruta=11 gera=5.94. This weight is in the range of the standard deviation of the average for beka 5.96+-0.04. Thus, instead of one weight of shekel of 640 pruta, they were using two standards: 21 gerah and double beka of 22 gera.

What is the reason for such strange weight system? One would expect the shekel of 640 pruta to be divided into 20 gerah of 32 pruta and beka to be exactly half shekel! We offer the following explanation. Since beka was used for the annual contribution to the Temple, it has to be a little bigger than a half shekel so that in case of underweight it will be not less than half shekel. If shekel would be 20 gera, beka should be 11 gera. But this is a very high surcharge of 10%. With gerah of 30 pruta and beka of 11 gerah, the surcharge on the correct half shekel is only 1/32. The shekel weight was made 21 gerah to match as close as possible the correct weight of 640 pruta. The sum of shekel and beka is 960 pruta or exactly the shekel of Torah. The nesef weight of 18 gerah is a nice rational part – 6/7 of the shekel weight. Together with two beka weights it produced the basic weight of 40 gera.

This weight system is naturally related to the units of volume. The basic unit of cup -the quarter of lug of 86.4 6 cm^3 of water weighs exactly 160 gerah. It can be expressed as a combination of 5 shekel and 5 beka or 4 nesef and 8 beka, but not as a combination of nesef, beka and the correct shekel of 640 pruta. The word nsf נסף in Hebrew means additional. Perhaps it was meant to be an additional weight.

What is the purpose of pym of 432 pruta? It is 4/5 of nesef, so that 5 pym and 8 beka are equal to the cup. On the other hand it is exactly 9 gerah of Torah, 9/10 of the half shekel of Torah. This weight is convenient for separation of the tithe-one tenth. If one subtracts one tenth from the basic unit of volume –the lug, one obtains 9/10x345.6 cm^3=311.04 cm^3=40 pym.

It is possible that the name pym פים in Hebrew hints to this relation. The letter מ has numerical value 40, the word פי means "times". Forty times this weight is equal to what is left from lug after separating the tithe.

We derived the above exact values by fitting the average weights. However the most frequent weights of shekel and beka are different from the averages. The first one is about 11.38-1.40 gr and fits 632 pruta of 11.376 gr. The second one is in the range of 5.8-5.85 gr and fits 324 pruta of 5.832 gr. The corresponding shekel of 632 pruta and double beka of 648 pruta deviate by 8 pruta from the correct shekel of 640 pruta. The weight of 648 pruta relates nicely to nesef of 540 pruta and pym of 432- it is 6/5 of nesef and 3/2 of pym. In the distribution of gerah one can recognize local cluster near the weight 0.64 gr and 0.47 gr. The first may represent the weight of 36 pruta of 0.648 gr and the second – the weight of 27 pruta of 0.486 gr. Both pym and nesef are multiples of these weights: pym=432 pruta=27x16=36x12 and nesef=540 pruta=27x20=36x15. The weight of beka of 324 pruta is also a multiple 324=36x8. The weight of the heavy gerah of 0.648 is represented by nine 3 gerah weights out of eleven (see the file) and can not be dismissed on statistical basis. It is equal exactly 10 barley grains of the British system. Hence, 700 of these gerah make one pound avoirdupois of 453.6 gr, 500 gerah make the Roman pound of 324 gr and 40,000 make the Attic talent of 1500 Attic tetra drachm. It is possible that there was an alternative weight system. This system, however, is not related well to the units of volume.

Finally, let us notice a surprising fact: the weight of beka of 330 pruta is equal exactly two Islamic dirham of 2.97 gr. Is it a mere coincidence? We think that it is not. The Jews exiled to Babylon carried with them their weights. The Babylonian exile existed continuously until the conquest by Arabs and after that. Umar Ibn al-Khattab who fixed the weight of dirham as 0.7 of the golden dinar, had Jewish aids (e.g. Kaab al Ahbar who showed him the location of the Temple) that could have given him the ancient Jewish standards. Since dirham is a synonym of a drachma which is a quarter of shekel, it was set equal to half beka. Gaonim which resided in Babylon knew about this identity. On the other hand, some of them (e.g. Rav Hilai Gaon) identified the beka weight they possessed with the sela of Yehuda in Mishna Trumot and the corresponding Yerushalmi. This lead them to the conclusion that lug is equal 100 dirham. The corresponding cubit of 45.64 cm was close to Roman and Byzantine standards and did not seem too small. Yet there is an inherent contradiction in this opinion. If dirham was the quarter of the shekel of 640 pruta, then the Islamic mitqal of 10/7 of the dirham could not be a quarter of the shekel of 768 pruta. The ratio 10/7 was probably meant to be an approximation of $(6/5)^2$. (Remark 39: According to Rif in the first version of his pages on Yiruvin 82b hundred mitqal are 144 dirham). Thus shekel of Gaoinim corresponds to double increase of the shekel of the first Temple by the ratio of 6/5. However, such double increase is not mentioned in Jewish sources.

17. Now we will bring evidence that the shekel of Torah was 17.28 gr.

a) Josephus writes in Antiquities (book 3:8:2) "And when he (Moses) had gathered the multitude together again, he ordained that they should offer half a shekel for every man, as an oblation to God; which shekel is a piece among the Hebrews, and is equal to four Athenian

drachmae". It is clear that Josephus talks about the shekel in the desert, not the shekel of his time.

b) Yonathan Ben Uziel translates the word "beka" in Genesis 24:22 and Exodus 38:26 as "drachmona". This word appears in Ezra 2:69 as drachmon of gold. Ibn Ezra explains there that drachmon is dinar of gold. The only golden dinar in the time of Ezra was the famous Persian daric. Its weight was close to Attic didrachma of 8.64 gr (see Smith dictionary). There are not many surviving coins. On the ANS site there are 45 coins over the period of years -500 -320 (out of them 7 double darics). Their average and also the most frequent weight is around 8.33 gr –see the histogram (the double darics are counted twice). This weight is exactly 10/7 of the most frequent beka stone weight of 324 pruta=5.832 gr. Notice the parallel between the Islamic mitqal and Persian daric. Both are based on 10/7 increase of the beka weight, but of different standards: beka of 330 and beka of 324 pruta correspondingly.

c) We already quoted in §12 Rashi's commentary on Exodus 21:32 that the weight of the shekel is four goldens. These are the golden coins of Constantinople as Rashi says by himself in Avoda Zara 11a. As we mentioned in §12, their weight in the time of Rashi was almost identical with the Attic drachma. Since Rashi's commentary refers to the shekel of Torah, this shekel is about the Attic tetra drachma. Yet, Rashi contradicts himself by identifying this shekel with half Cologne ounce. It is possible that Rashi carried two traditions. The first one was based on identification of beka of the Torah with the Persian golden daric and on the knowledge that the golden coins of Constantine were about half of these darics or of the Attic didrachma. The second one was based on the weights of the Tyrian shekel.

d) It is written in Nechemia 10:33 "We also lay ordinance on ourselves, to charge ourselves yearly with the third part of the shekel for the service of the house of our God". Why they gave third part of the shekel instead of one half? What kind of coin was it? The only silver coin that existed in time of Nechemia was Persian silver siglos (besides the small Yehud coin-see below). On the site of ANS there are several hundreds of them. On the histogram is shown the distribution of the weights of 221 coins. The average weight is 5.50, standard deviation 0.1. Without exceptional coins, average 5.52 (out of 181), standard deviation 0.05, standard deviation of the average is 0.04. The most frequent weights are in the range 5.52-5.56. Evidently siglos is 2/3 of the daric. Nechemia calls them third of the shekel of Torah since the weight of double daric was close to the weight of the shekel of Torah. Persians did not have a silver coin of such weight and hence it is impossible that Nechemia was referring to the Persian shekel. On the other hand, siglos was almost equal to the half shekel of the First Temple. Therefore, in the first years after the return from the exile the Jews were contributing the same amount as before the destruction of the Temple. Though siglos was a little less than half shekel, they were apparently adding to it a small coin -kalbon.

It is known that at the beginning of the Second Temple period during the Persian rule there were in Yehuda small silver coins, so called Yehud coins. Many of the found coins are listed on the site. These coins have Yehud inscription but different images and different weights. Five of their types have sufficiently many specimens for statistical analysis. The heaviest type has images of portrait and owl. Here is the histogram of their published weights. This histogram is very similar to the one of gerah. Because of wear, the weight of coins is a little less than the weight of gerah stones. The biggest cluster is around 0.5 gr and is identified

with gerah of 30 pruta=0.54 gr. The second cluster is around 0.45 gr and is identified with gera of 27 pruta=0.486 gr (recall that pruta according to our claim is 0.018 gr). The third cluster is around 0.61 gr and is identified with gera of 36 pruta=0.648 gr. If one was adding to average siglos of about 5.5 gr a gera of about 0.5 gr, the resulting weight was about 6 gr. This is exactly the half shekel of the First Temple of 320 pruta=5.76 gr plus kalbon of 1/24.

The second type has images of lily and falcon. Here is the histogram of their published weights. The average weight is 0.34 gr and the most frequent one is 0.36 gr. We identify this weight with 20 pruta=0.36 gr. This is exactly 1/48 or pondion of the shekel of Torah of 17.28 gr. It is equal to half of Attic obol. According to Mishna Shekalim 1,7 kalbon was half of meah, where meah is 1/24 of shekel. This is the opinion of the Rabies. Rabbi Meir says that kalbon is meah. As we mentioned at the end of §15, the word gerah is translated as meah. Hence addition of gerah to the basic coin of half-shekel (the Persian siglos) corresponds to the opinion of Rabbi Meir and the addition of pondion to the opinion of Rabies.

The third type has images of the head of Persian king and a falcon. Here is the histogram of their published weights. The average is 0.28 gr and the most frequent is around 0.30 gr. We identify this weight with 16 pruta or 0.288 gr. This is the pondion of the shekel of the Second Temple of 13.824 gr.

The fourth type has images of male head and lynx. Here is the histogram of their weights. The average weight is about 0.23, the most frequent is 0.20-0.21 or 0.24-0.25 gr. These coins should be compared with the fifth type that has inscription "Hezekiah the governor" and images of head and owl. Here is their histogram. The average and most frequent weight is 0.20 gr. Hence in the fourth type we should consider the two most frequent weights as two standards. The weight 0.24-0.25 is identified with pondion of the shekel of the First Temple, i.e. 11.52/48=0.24 gr or 1/48 of double beka of 660 pruta equal to 0.2475 gr. The weight 0.20-0.21 corresponds to 12 pruta or of 0.216 gr. The last weight is the smallest kalbon that, when added to siglos, exceeds the weight of the half shekel of 320 pruta of the First Temple Indeed, the "theoretical" siglos is 2/3 of the daric, while daric according to our calculation is 10/7 of the beka of 324 pruta. Hence the "theoretical" siglos is 308.57 pruta or 5.554 gr. The rounded number of 308 pruta of 5.544 gr lies in the middle of the range 5.4-5.6 of the most frequent siglos (see histogram). The sum of it and 12 pruta is exactly 320 pruta.

The Yehud coins existed also in the beginning of the Hellenistic period. The type with the largest number of specimens has the image of Ptolemy I with eagle. The most frequent weight of coins is 0.20 gr as above. The type with the second largest number has the image of bird. Their weights cluster around 0.36 gr as in the second type above.

Thus, we demonstrated that the five types of weights of the Yehud coins were: gera, three standards of pondion and a minimal kalbon of 12 pruta. All were used as kalbon to the Persian siglos for the annual contribution to the Second Temple. This contribution was at least half shekel of 320 pruta. Since it is called by Nehemia one third of the shekel, the original shekel of Torah was 960 pruta or 17.28 gr.

18. We will bring now additional evidences for our claim from Mishna and Talmud.

a) The Mishna Sheviit 1,2 says "what is the field of trees- three trees in beit seah (2500 sq. cubits); if they are fit to produce kikar of devela (dried figs) of sixty Italian maneh, then the whole beit seah is ploughed for them". On the first glance, Italian maneh is the Roman pound. The Mishna calls 60 pounds kikar. The holy kikar of Torah is 3000 shekels, the secular is 1500 shekels. Since Roman pound is 324 gr, the corresponding shekel is either 6.48 or 12.96 gr. Both weights do not fit any of known shekels. The kikar was an equivalent of talent. However Roman talent was not 60 pounds but 100 pounds. Thus the first explanation of the Mishna is problematic. We already mentioned in the beginning of §9 that Rome was called the Italy of Greece and that Roman and Greek standards of weights were related by factor of 4/3. Hence it is possible that Italian maneh is actually the Attic mina of 432 gr. The Attic talent was indeed 60 mina. If this talent was equal to the secular kikar of 1500 shekel, then the corresponding shekel is exactly 17.28 gr.

b) Yerushalmi in Sanhedrin 9a brings the discussion between Antinonus and Rabbi Yohanan Ben Zakai. Antinonus claimed that Moses was either a thief or did not know how to count. The Children of Israel contributed to the Mishkan half shekel each one (total 603550). Moses counted them as 100 kikar plus 1775 shekel. Now, Antinonus asks: if the kantar was 100 litra then Moses stole one sixth. If it was 60 litra, then Moses stole one half. It is clear that Antinonus understands that kikar of Torah is an equivalent of the talent and calls it kantar (the word is apparently derived from centum-hundred). The Roman talent was 100 Roman pounds. Antoninus counts 25 shekels as litra, a talent 2500 shekels, 100 talents 250,000 or 5/6 of the total of 300,000 shekels. Next he tries the Greek standard. Again litra is 25 shekels but the talent is 60 litra or 1500 shekels and 100 talents are 150,000 –half of the total. Rabbi Yohanan Ben Zakai's answer is that the holy kikar was double, 3000 shekels. From the answer we understand that the standard of Torah was the Greek one, not the Roman.

c) The Talmud in Kidushin 12a brings a discussion about the weight of pruta. Rav Yosef thought that that the value of pruta is not fixed. He brings Beraita (external Mishna) that says "go and count how many prutot are in two sela, more than two thousand" (Remark 40: The Beraita talks about a person who enjoyed from the Sanctuary by pruta and has to bring a sacrifice which costs two sela, much more than he enjoyed). The Talmud brings another version of the Beraita which says not "more than two thousand" but "close to two thousand". Still the amount of prutot in two sela is only 1536. The Talmud explains that since this amount is more than a thousand and half, the Beraita calls it "close to two thousand". It is a strange story. First, Rav Yosef had a wrong version of the Braita. Second, the Braita itself is extremely non exact. Third, the price of the sacrifice is two shekels of Torah which is supposed to be 1280 pruta, not 1536 pruta. We offer the following explanation of the Talmud. Rav Yosef knew that the shekel of the first Temple was 2/3 of the shekel of the Torah, but he thought that this shekel was divided into 960 prutot as the shekel of the Torah. This proves that the pruta can be decreased. Rav Yosef understands that Braita talks about the sela of the second Temple which is 6/5 times larger than the shekel of the first Temple. Hence two sela are equal to 2304 prutot, more than two thousand as says the Beraita. As for the price of the sacrifice, he understands that it was set equal two shekels of the current time. As the Talmud brings the version "close to two thousand", Rav Yosef can explain that it talks about the first Temple period (or beginning of the second Temple) when the shekel was 960 prutot, hence two shekels

are 1920 prutot- close to two thousand. The Talmud suggests instead that the Beraita talks about the sela of the second Temple and counts it as 768 prutot. Hence the shekel of the first Temple was 640 prutot and the value of the pruta did not change. The Talmud also understands that the price of the sacrifice is two shekels of the current time.

19. The relation between the weight and volume can be inferred from the book of Ezekiel. It is written in Ezekiel 4: 10 "And the food which thou shalt eat shall be by weight , twenty shekels a day". A standard ration of a man according to the Talmud is two meals a day during the weekdays. As for the size of the meal, there are two basic opinions. One is –four eggs of food and the second three eggs. Both opinions are brought in Shulchan Aruch Orech Haim 368, 3. The food is either bread or wheat grain or wheat flour. (Remark 41: There is no contradiction between these three possibilities. From a volume of grain, after removal of bran one obtains roughly the same volume of flour and then the same volume of unleavened bread). The specific weight of wheat grain is about 0.77. The specific weight of flour depends whether it was pressed or falling free (it could range from 0.8 to 0.55 –my own measurements). From weights brought by Rambam it appears that the density of flour is 2/3 (about the average between pressed and free falling flour). Thus the maximal weight of two meals is 8 eggs of wheat and the minimal is 6 eggs of flour. (Remark 42: Since the specific weight of bread is not known, it cannot be used for metrological purposes). If we approximate the specific weight of wheat by 0.75 then 8 eggs of it weigh exactly as a lug of water. The minimal weight is 2/3 of the lug. Ezekiel was eating a mixture of "wheat and barley and beans and lentils and millet and spelt" (Ezekiel 4:9). The caloric value of them is about 350 calories per 100 gram and so is the caloric value of whole grain flour. Hence we will equate the weight of food of Ezekiel with the above standard meals. The maximal weight gives the equation 20 shekels=lug of water. This is exactly the relation between the shekel of Torah of 17.28 gr and the lug of 345.6 cm^3. The minimal weight corresponds to the shekel of 2/3x17.28=11.52 gr. This is exactly the shekel of the first Temple. If 20 shekels of the first Temple are equal 2/3 of a lug of water then 25 shekels of the second Temple are 25/20x6/5=3/2 times more, or exactly lug of water. This is the opinion of Rashi (see §12 above). (Remark 43: The other two possibilities: 8 eggs of flour and 6 eggs of wheat give equations: 20 shekel=8/9 lug and 20 shekel=3/4 lug. They do not fit any known opinion about relation between Torah weights and volumes).

Let us check the caloric value of the food eaten by Ezekiel. The maximal weight gives about 20x17.28x3.50≈1200 calories. The minimal weight gives about 800 calories. Ezekiel was eating this food for 390 days. The minimal caloric intake (basal metabolic rate-see calculator) for a men age 50 weighing 60 kg and 160 cm high is about 1350 calories. On the other side, in the siege of Leningrad during long period workers were given daily 600 gram of bread and their dependants 400 gram (in the first winter the ration was much less but about 600,000 died). The caloric value of 400 gram of coarse bread is about 800 calories. Since Ezekiel was lying on his side and not moving, he could have survived this diet (Remark 44: though 6 eggs of food is considered by the Talmud as a normal diet, it does not include fruits and vegetables that a man eats in addition to bread). His water intake of 1/6 of hin or two lug of 691.2 cm^3 was also below recommended minimum of about 1 liter.

We think the text in Ezekiel suggests both of the above solutions. The Talmud in Yiruvin 81a comments on the verse in Ezekiel 4:12 "And you shalt eat it like barley cakes". Rav Hisda says "by measures" (the word "measures" שעורים spells as "barley"). Rashi explains there – you should eat by weight as in hunger. Rav Papa says that the form of the bread should be unpleasant, as bread of barley and not of wheat. The first commentary means that Ezekiel actually suffered hunger. The second may mean that it was only appearance of hunger.

20. Now we will bring several hints from Torah.

a) In Numbers 7 each of the twelve heads of tribes contributed a golden spoon of ten shekels full of incense. On the other hand it is written in Talmud Kritot 6a that incense was burned (on the inner altar) twice a day, half of maneh in the morning and half of maneh in the afternoon. The maneh referred by Talmud is of 25 shekels of the second Temple. There is apparently relation between the vessel and it content. The Talmud in Nida 21a says "said Abai in the whole world (a vessel) containing lug is made from maneh, two lug is made from two maneh, but the cup of Tiberia even containing two lug is made from one maneh". As we have shown the lug of water was equal to the maneh of the second Temple. Abai is saying that there is a general rule that the weight of a vessel is equal to the weight of his content. If this relation applies to the golden spoon, then the incense was weighing ten shekels of Torah. The amount of incense on the other hand should be the standard amount burned in the Temple. Hence, we obtain the equation: 10 shekels of Torah are equal 12.5 shekels of the second Temple.

The heads of tribes also brought "one silver dish the weight of which was a hundred and thirty shekels, one silver bowl of seventy shekels after the shekel of the sanctuary: both of them full with flour mingled with oil for a meal offering". The standard meal offering was omer=3/10 of seah. Two vessels contained two offerings of 3/5 of seah or 72/5 lug. Their combined weight was 200 shekels. The meal offering consisted of wheat flour. With specific density of flour equal to 2/3 of water we obtain equation: 200 shekels=48/5 lug or 20 shekels=0.96 lug. It is possible that the overweight of 1/24 hints to the amount of chala 1/24 of the dough that should be given to the priests (Remark 45: the weight of oil was apparently not counted since it was absorbed in the flour).

b) Josef put his silver bowl in the bag of Benjamin not only to accuse him in theft and pretend as to take him as a slave but to hint to his brothers that he was stolen by them and sold into slavery. It is written in Genesis 37:28 that Josef was sold for twenty silver. Yerushalmi Shekalim 9b says, "since the firstborn of Rachel was sold for twenty silver, hence everyone will redeem his firstborn son for twenty silver". Since the firstborn is redeemed by five shekels, these pieces of silver were dinars. The volume of a standard drinking bowl or cup is a quarter of lug. According to the rule mentioned above, the weight of the bowl was the same as its content, i.e. five shekels of Torah. Thus Josef hinted to his brothers the exact amount he was sold for. When a father gives the priest five shekels for his firstborn, he also makes a blessing on the cup of wine of the same weight.

c) There is another hint related to Josef. The Talmud (Shabat 10b) states that because of two shekels weight of garment Jacob gave to Josef, more than to the rest of the sons, our forefathers

went into exile in Egypt. What was this garment? Midrash Tanchuma, Ch. Vaeshev 4 says that Jacob gave to Josef a strip of purple. There is a commandment of the first of fleece that is apparently related to this garment. The minimal amount of wool one should give to a priest is five sela of Yehuda, enough to make a small garment (Chulin 135a). The Talmud in Chulin 138a concludes that the small garment is the sash of the priest. Apparently, Jacob made for Josef a sash of purple on top of his coat and caused by this envy of his brothers. Hence the five sela of Yehuda should be equal to two sela of Torah given to Josef. We saw in §13 that sela of Yehuda was a half shekel of the second Temple. Thus the shekel of Torah is 5/4 of the shekel of the second Temple and 3/2 of the shekel of the first Temple.

d)   The basic heaviest unit of weight of Torah is kikar. There was a holy kikar of 3000 shekels and secular kikar of 1500 shekels. The latter is identified with the Attic talent (see §18 b). The origin of the name kikar is not clear. In Hebrew a loaf of bread is also called kikar. This is in supposed to be a food for two meals. In the desert, the children of Israel were given instead a portion of manna. The daily amount of manna was equal omer or 3/10 of seah. This was the substitute of the loaf of bread (Remark 46: The volume of omer is 5.4 times bigger than standard kikar of 8 eggs. Why? The taste of manna depended on the imagination of the eater. The ideal bread was the showbread. Each one of the twelve showbreads was made from two omer of flour and had a volume of 10x5x1=50 cubic palms. Hence the ratio of its volume to the volume of flour was 5.14.., close to the ratio of 5.4. If one was eating manna and wanted to imagine the taste of the showbread, he had the feeling that he is consuming about 8 eggs of flour (more exactly 8 eggs and an olive of 1/2.5 of an egg for the third meal of Shabbat)). On the other hand there is an obvious connection between money and food. A surprising coincidence: omer full of silver weights about the same as the secular kikar of 1500 shekels. Here is the calculation. The density of pure hammered silver is 10.5 of water. Omer of silver weighs as 10.5x3/10 seah of water. Since seah equals 24 lug and lug weighs 20 shekels of Torah, the omer of silver weighs 1512 shekel. In reality the silver coins are made from an alloy, otherwise they are not durable. The finest standard of coins used in the second Temple was the Tyrian one. It contained on the average 94.56% of silver (Remark 47: see Metrology of Roman Silver Coinage, British Archeological Report 5, 1976, p.58 by D.R. Walker). Taking the rest to be copper (density of 8.96) one obtains the average density of 10.416 or 10 5/12. The corresponding weight of the omer of silver is exactly 1500 shekel.

e)   The total amount of gold used in the Tabernacle was 29 kikar and 730 shekels (Exodus 38:24). The covering of the arc was 2.5 by 1.5 cubits (Exodus 25:17). According to Talmud (Succa 5a), the covering was a palm thick. Thus, its volume was 135 cubic palms or exactly 200 lug. Since it was made of pure gold, its weight was 200x20x19.3=77200 shekels or 25 kikar and 2200 shekels. We should reserve also kikar for Menorah. The amount of gold for the rest of the vessels is not specified, so it could be very small. For example, the wood could be covered by few microns of gold. Thus, the total amount of gold could suffice. But if the shekel of Torah was the same as the shekel of the Second temple (at most 17.28/1.2 gr), while the cubit was 48 cm, then the covering would weigh 77200x1.2 shekels, which is more than total amount of gold. (Remark 48: Ibn Ezra in his commentaries on Exodus 38:24 writes that "we received from our forefathers that kaporet (the cover) was one kikar". Notice that the volume

of the cover was 135 cubic palms exactly as the volume of the sockets of the boards. The latter were made from one kikar of silver. The silver needed to fill this volume is about 14 times more. The sockets thus were hollow. We then suggest by analogy that also kaporet was hollow. May be it had the same volume of gold as kikar of silver but its weight was greater according to the higher density of gold).

f) In the paper The form of the Menorah we calculated the form of the Menorah based on its weight of 3000 shekels of 17.28 gr and its known form. In particular, 2000 shekels were spent on the body of the Menorah and 1000 shekels on its fixtures. The weight of the body is determined almost uniquely from its form and the minimal standard diameter of the stem and the branches- a thumb. With a cubit of 48 cm and shekel of Torah being equal to the one of the first Temple, all gold would be spent on the body and nothing would be left for the fixtures.

21. The devaluation of the shekel from its original value can be inferred from the price of land. Abraham bought the Cave of Machpela and the surrounding field for 400 shekels. According to Torah (Leviticus 27:16) the price of beit kur of 75,000 sq cubits is fifty shekels. Hence, the size of the field of Machpela was 600,000 sq. cubits (Gaon from Vilna in Qol Eliyahu,Ch. Haei Sara relates this area to the standing place of 600,000 men). On the other hand, David bought the Temple Mount from Aravna the Yebusy for 50 shekels of gold (Shemuel II, 24:24) which are equivalent to 600 shekels of silver (Chronicles I, 21:24). The area of the existing Temple Mount (surrounded by the red line) is equal 600,000 sq cubits of 48 by 49 cm. (Remark 49: The calculation is based on exact measured coordinates of many points at the exterior walls of the Temple Mount (the black points on the red line). The equality is satisfied with relative error of about 1/10,000!). (The essence of the cubit of 49 cm was explained in §6). Apparently David bought the area of the expanded Temple Mount to accommodate 600,000 men. (Remark 50: The actual exterior walls were however built in different times). The price of this land according to Torah should be 400 shekels. Why David paid 600? (Remark 51: We are not talking here about market value. The purchase was a symbolic act, though completely real). If the shekels of David were the ones of the first Temple, then he paid exactly 400 shekels of the Torah.

Jacob also bought a piece of land. "And he bought the piece of land on which he had spread his tent, at the hand of the children of Hamor, Shekhem's father, for a hundred kesita" (Genesis 31:19). What is kesita? The Talmud is Rosh Hashana 26a says "Said Rabbi Akiva: when I was going to Africi, they were calling to meah- kesita". The Talmud asks "for what sake we should know it?" and answers- "to know that hundred kasita of Torah are hundred danki" (danka is one sixths of dinar or 1/24 of sela). Rabbi Akiva is talking about meah of his time which was 1/24 of the Tyrian shekel. Hundred kasita are thus 100/24 shekels of the second Temple. What is the size of land Jacob bought? He spread there his tent and built an altar. It hints to the size of the court of the Tabernacle. This court was 50 by 100 cubits. According to Torah the price of such area should be 5000/75000x50 =10/3 shekels of Torah. If we equate these numbers, we obtain the relation: shekel of Torah is equal 5/4 of the shekel of the second Temple. This is exactly the relation we claim. The shekel of the first Temple was 20 meah or

kesita, or 2/3 of the shekel of Torah. The Talmud in Shabat 33b comments on the verse "and he camped before the city" (Genesis 31:18) and says that Jacob made a favor to the people of Shem (the word "camped" in Hebrew has also a meaning –"favor"). What is the favor? "Says Rav-he introduced to them a coin". Thus, Jacob invented the coin kesita. With this unit he could express the price of the piece of land of 3 1/3 shekels of Torah as a round number of 100. (Remark 52: Though Jacob paid the standard price, the large amount of new units apparently impressed Shekhem. According to Midrash he said to Dina: your father spent a lot of money for a small piece of land. If you will marry me you will get the whole city and all its fields). Unlike gera that is 1/20 of the shekel of Torah, kesita is 1/30 of it. This kasita became the meah or gera of the shekel of the first Temple. Thus the reduction of the shekel was actually hinted by Jacob.

22. The change of the weight of the shekel is implied by the verse in Ezekiel 45:12 "And the shekel shall be twenty gera: twenty shekels, five and twenty shekels, fifteen shekels, shall be your mane". Tosfot in Baba Batra 90a brings two interpretations of this verse. One is of Rabenu Tam and the second of Rabenu Yizhaq. According to both of them the shekel of Ezekiel was the shekel of the first Temple (which is thought to be also the shekel of Torah). Rabeinu Tam says that in the time of Sages (after Ezekiel) the shekel was increased by factor 6/5 from 20 to 24 gera. The mane of Sanctuary in the future will be 60 shekels but of 24 gera, $(6/5)^2$ times more than the maneh of Sanctuary of the first Temple. Rabenu Yizhaq says that Ezekiel added four gera to the shekel. His maneh of 60 shekels of 20 gera is the mane of Sanctuary of the second Temple of 50 shekels of 24 gera. The Talmud brings this verse (there) and learns from it three things: the mane of Sanctuary was double; the measures may be increased but not more than by one sixth; one sixth is taken from outside (i.e. the difference between 60 and 50 is 1/6 of 60). It is clear that the Talmud refers only to the increase of the mane from 50 to 60 shekels, not to the increase of the shekel. As for the division of 60 shekels into sum of 20, 25 and 15, Targum (translation) of Yonathan says that 20 is a third of the mane, 25 is mane of silver (or money) and 15 is a quarter of mane- together 60 which will be the great mane of Sanctuary. Tosfot says that the Targum does not fit the opinion of Rabenu Yizhaq. If Rabenu Yizhaq will interpret the Targum that a third of mane of present (of the second Temple) was 20 shekels of the past (in the first Temple), then likewise the secular maneh of silver of present (of 25 shekels of 24 gera) was 25 shekels of the past- which is wrong. In our opinion, there is also a difficulty with the interpretation of Rabenu Tam. Ezekiel was talking about the maneh of the future Third Temple. If the shekel of it will be 20 gera, then it will be decreased relative to the shekel of the second Temple.

Our interpretation of the verse is following. The shekel of 20 gerah is the original shekel of Torah of 17.28 gr. The future maneh of Sanctuary will be 60 such shekels. This weight is equal to three lug of water or wine. This is the minimal amount of drink offering (Exodus 29:40) and the basic measure of water offering on Succot (Succa 4, 9). The units of 25, 20 and 15 served as mane in the past. Twenty five shekels of 17.28 gr was the secular maneh in the desert. This is also the Attic maneh. Targum calls it "mane of silver". Twenty shekels of 17.28 gr is the lug of water or wine. This was the maneh of the 25 shekels of the second Temple. This is the standard maneh of Mishna and Talmud. Fifteen shekels of 20 gera of the **first**

Temple was the litra of Yerushalmi Trumot (see the beginning of §13). Thus the verse in Ezekiel refers partially to the shekel of the first Temple. In the future, fifteen shekels of 17.28 gr will be quarter of the mane of Sanctuary and 20 shekels will be a third of it, as says the Targum. The future (and the past) shekel will be a result of double increase of the shekel of the first Temple by factor 6/5 with an addition of kalbon of 1/24. (Remark 53: if one starts with the average double beka of the first Temple of about 12 gr (see the [distribution](#)) then the future shekel is exactly $(6/5)^2$ of the past). Thus the final shekel is obtained by combining both opinions of Rabenu Tam and Rabenu Yizhaq. First the shekel is increased by adding to it four gera. Then the mane of 50 such shekels is increased to maneh of 60 shekels. Finally this maneh is divided back into 50 shekels.

23. Our claim that the shekel of Torah was reduced to 2/3 of its original value apparently contradicts the statement of Shmuel in Baba Batra 90b, that the measures may not be changed at once by more than one sixth. The Talmud also learns in Bechorot 50a that the shekel can be increased but not decreased. There is also no tradition that the weight of the shekel was decreased upon entering the Land of Israel.

As for the first objection, the Talmud there brings reasons why the measures should not be changed by more than one sixth. One is that the prices may go up too much. Another is that some people will sell or buy mistakenly for price that differs from the market value by more than one sixth. According to Torah such purchase is cancelled. The third is that some merchants may loose not only the profit (a normal profit is about one sixth) but even the capital. The Talmud rejects these reasons since then even a change smaller than one sixth should not be allowed. The Talmud says finally that Shmuel learned his rule from the verse in Ezekiel 45:12. Now, when children of Israel entered the Land of Israel, they did not have or almost did not have a market economy. Their food was the heavenly manna, their clothes were growing with them. They also did not have real estate. Hence the above reasons do not apply to them even when the change is greater than one sixth. (Remark 54: It could be a problem if somebody sanctified a women with a reduced pruta while she thought that this is non-reduced one. Yet according to our claim the pruta did not change). The verse in Ezekiel says that despite the above reasons, it is allowed to change measures by one sixth. In a situation where there are no reasons, this verse does not apply.

As for the second objection, the Talmud actually says that the number of gera in shekel cannot be less than twenty; it does not say the weight of gera cannot be reduced.

As for the third objection. The Talmud in Yoma 80a brings Beraita (external Mishna) "The "shiurin shel onashin" (measures of punishments) are rules given to Moses in Sinai (i.e. they are not written explicitly in Torah). Others say the court of Yavez has established them". The Talmud replies that a prophit can not make new rules. Finally the Talmud explains- they were forgotten and were returned and reestablished. Yavez is the name of the judge Atniel Ben Kanaz, step brother of Kaleb. The Talmud in Tmura 16a brings Beraita "Thousand and seven hundred "kal vehomer" (light and strict), "gzirot shavot" (equal cuts) and "dikdukei sofrim" (delicate observations of the counters) were forgotten in the days of mourning of Moses". Said Rabbi Abahu: dispite that, they were returned by Atniel Ben Kanaz using his sharp reasoning

(Remark 55: my English translation of Talmud is a very crude approximation of the original). Apparently these two pieces of Talmud are related. The forgotten measures of punishments are part of the 1700 forgotten things. Measures of punishments are for the example the minimal volumes of forbidden food, like the size of an olive, of forbidden drink, like the quarter of lug. However, these are mentioned in Talmud Suca 5b and are called simply "measurements". We suggest the following explanation. The word "onesh" (punishment) is mentioned in Torah twice: in Exodus 21:22 and Deuteronomy 22:19. In both cases it means monetary payment. In the first case the payment depends on the damage, in the second case it is a fixed fine of 100 shekel. Perhaps "measures of punishment" is the precise amount to be paid? (Remark 56: The phrase "measures of punishments" does not refer only to the second case but to all cases of a fixed fine, like 30 shekels of a slave killed by a bull or 50 shekels in case of a rape or seduction). But it is written explicitly in Torah and is not a rule given orally! In addition, how could this rule be forgotten? The answer is: it is the weight of the shekel which is not written. However, the Children of Israel learned what the weight of the shekel is when they contributed half shekels for the first time! How could they forget it? Answer: we will show in §24 that the weight of shekel was given as a number of grains of manna. This relation is not written in Torah. It was "forgotten" in the days of mourning after Moses because the manna stopped to fall with his death. However, they had vessels in the Tabernacle of fixed weight like the Menorah of 3000 shekels or golden spoons of 10 shekels! They could weigh shekels against these vessels! A possible answer is that it is not allowed to use vessels of Sanctuary for secular purposes. But they certainly had stone weights in the desert as we learn from the verse in Deuteronomy 25:13! Even if these weights were not 100% exact, why they reduced the shekel by 1/3?

The Talmud in Suca 5b says that the measures of Torah are implied by the verse "a land of wheat and barley and vines and fig trees etc." (Deuteronomy 8:8). This means that the measures of Torah should be related to the fruits of the Land. Hence the shekel which was defined in terms of manna, had to be redefined in terms of fruits mentioned in the verse. The most natural definition is the number of wheat or barley grains. Rambam wrote in his commentary on Mishna Bechorot 8,7 that he received from his father that received from his father and his grandfather that the piece by which we estimate the shekel (before that Rambam explained that shekel is equal 24 dracmon and drachmon is equal 16 pieces, total 384 pieces) is the barley grain and he (Rambam's father) does not know any reason for it". But it is known that the weight of barley or wheat grain depends on many factors: the kind, the field, the year, is it an average weight or taken from the middle of the ear as in English standards. May be this was an oral rule given to Moses, how to translate the grains of manna into grains of barley or wheat. We think that this knowledge is hinted in the words "dikdukei sofrim". Sofrim were the great scholars of Torah which were counting numbers and were keeping quantitative relations. The particular knowledge here was the number and essence of the corresponding grain. This knowledge was forgotten and Atniel Ben Kanaz recovered it. But as he recovered it, it was possible to change the weight of the shekel because of several considerations. For example, it is possible that they wanted to keep a fixed price of wheat: four seah for a shekel. This price, however, depends on the rate of exchange between wheat and silver. As long as they were in the desert and there was no trade, the amount of silver they possessed was enough

to sustain a heavy shekel. As they entered the Land of Israel and became a "normal nation" they needed more silver. Since the amount of it was limited, they would have to reduce the weight of shekel. Another consideration is the relation between weight and volume. In desert they used the relation 20 shekels equal lug of water. In the Land of Israel, they may have replaced water by wheat flour with specific gravity of 2/3. They also may have adopted for gera the weight of kasita of Jacob (see §21). (Remark 57: We will show in the next paragraph that gera of Torah consisted of 24 grains of manna arranged in a box of 12x12x8 mm. The new gera consisted of 16 grains of manna arranged in a box of 8x8x12 mm).

All these are of course conjectures. But it is evident that the measures were forgotten and were recovered. And when recovered, they could have been adjusted to a new reality.

24. This paragraph constitutes a separate article named "**The form of the manna and the weight of shekel**".

Manna was the heavenly bread of the Children of Israel in desert. Each one was getting the same constant volume of manna, omer per day. Thus manna defined the unit of volume and hence the unit of length. It also determined the time: on Friday there was a double portion of manna while on Saturday there was no manna. The Talmud in Brachot 27b learns that the manna was melting at the end of first three hours of the day. This was a sign of the end of time of calling "Shema Israel". We have shown in §20 d) that the kikar of 1500 shekels of Torah was equal to the omer full of silver. In this article, we will show how the manna defined the monetary system of Torah. (Remark 58: the resemblance of Hebrew words: מן (man) , מנה (maneh), ממון (mamon-capital) and the English "money" derived from Latin "moneta" as well as the name of the Greek deity "Hera Mone" (see site) which combines the names of the basic units "gera" and "man" is not a chance coincidence).

The key to the relation between manna and money is hidden in the mystery of the weight of the sockets. The sockets (in Hebrew אדנים- adanim) were kind of shoes at the basis of the boards of the tabernacle. The boards were 1.5 cubits wide and a cubit thick. Each socket was covering the lower cubit of half a board. Hence his exterior dimensions were 3/4 by 1 by 1 cubits or 4.5x6x6 palms. According to the Beraita of Melechet HaMichkan and the commentary of Rashi in Exodus 26:17, the arm of the board of 1.5x3x6 palms entered the socket. Thus the volume of the socket was 4.5x6x6-1.5x3x6=135 palms^3 =69120 cm^3. These sockets were made from the half-shekels collected from all Children of Israel, for each socket a holy talent of 3000 shekels of 17.28 gr or 51.84 kg. However the amount of silver necessary to fill the above volume is 720 kg, i.e. 125/9 times larger (Remark 59: we assume that the specific gravity of silver is 10 5/12 as in §20 d) . This implies that the casting of the sockets had 92.8% empty space.

What was filling the empty space? Recall the correspondence between the loaf of bread and the loaf of silver. The volume of 135 palms^3 is exactly 1200 eggs. Recall also that according to our first interpretation of Ezekiel 4: 10 in §19, the weight of 8 eggs of wheat is 20 shekels. If one fills the volume of the socket with grain, its weight will be 1200/8x20=3000 shekels! Now the hint is clear. The casting form for the socket was filled up with a grain. On

the other hand Children of Israel weighed the same weight of silver. The silver was then put into the form and filled up the empty space between the grains. Since their bread was not grain but manna, we conjecture that they filled the form with the grains of manna and put the silver into empty spaces. Then the whole cake was baked under the melting temperature of silver 961.9 deg. But this is impossible since the manna was melting "when the sun grew hot" (Exodus, 16:21)! It says also in Numbers 11: 8 about the manna "And the people went about and gathered it and ground it in mills or beat it in a mortar and boiled it in a pot". Yet our sages comment on this verse in Sifri (Ch. 89) that manna was never grounded or beaten or boiled but its taste was changing according to how one would like to prepare it. From this, one can learn that it was impossible to affect the manna by physical means. However, when put in mouth it would acquire the taste and consistency according to ones imagination. It was melting outside not because of the heat of sun but because it signaled to the Children of Israel the last time of saying "Shema".

Another property of manna is that it had a constant volume. It says in Exodus 16:17-18 "And the children of Israel did so and gathered some more some less. And when they did measure it was an omer, he that gathered much had nothing over, and he that gathered little had no lack etc." Actually, some made wrong measures and tried to prove that the manna had not a right volume. That is why they were attacked by Amaleq immediately after they cheated on measures. Therefore the warning "Thou shalt not have in thy bag divers weights, a great and a small etc" is followed by the warning "Remember what Amaleq did to thee" (Deuteronomy 25:17). If manna would be soft, one could have pressed it and reduced its volume. Also necessarily it had a unique packing. If the grains of manna were round, one could rearrange them to get a different volume. For example, put a ball above a ball to increase the volume or put it between three lower balls to decrease the volume. Hence the grains should have a form of a box. But we know that "manna was like coriander seed" (Numbers 10:7). The Talmud (Yoma 75a) explains that it was round as the coriander seed. Indeed, it the grains were like boxes they would cut the mouth with their sharp corners.

Yet there is a very special form that is simultaneously a box and a ball. This is the intersection of a box of 4x4x3 with a ball of diameter 5. Since $3^2+4^2=5^2$, the ball touches the middle points of the four edges at the top and at the bottom of the box. (Remark 60: This was the form of the ten bases of brass (Kings I, 7:27) 4x4x3 cubits. Built into it was a sphere of diameter 5 with a center in the middle of the base. The capital of the sphere protruding through the upper face of the base was cut at the height of half a cubit above the face, forming a circle (this is the round compass in verse 35). The radius of this circle was 1.5 cubits (this is the mouth of the capital in verse 31). From this one deduces that the diameter of the sphere was indeed 5). The volume of the intersection is equal to the volume of the ball of diameter 5 minus the volume of 6 caps protruding trough the faces of the box. For ball of radius r, the volume of a cap of height h is $\pi h^2(r-h/3)$. We have 4 caps of height 0.5 and 2 caps of height 1. Thus the total volume is

$$V=\pi/8 \times (4/3 \times 5^3 - 4(5-1/3) - 2 \times 2^2(5-2/3)) = 44.5059$$

versus the volume of the box 48. The difference of these two volumes is 3.4941 and constitutes 7.279% of the volume of the box. Compare it with the ratio 9/125=7.2% of the volume of the silver versus the volume of the socket. They agree up to 99%!

It remains to find the length of the scale for the box. Recall that the walls of the socket are 1.5 palm=12 cm thick and 6 palms=48 cm high. In order to fill it with boxes of size 4x4x3, they should be of the size 4x4x3 cm or an integer fraction of it. Since the socket has the volume $69120 cm^3$, there are 1440 such big boxes filling the socket. However we wish the division into boxes to fit the division of the socket into 3000 shekels. Since the ratio 1440/3000=12/25 has denominator 5^2, the big boxes should be divided into 5^3 small boxes of the size 8x8x6 mm. But the Torah compares the form of manna with coriander seed (Exodus 16:31) which is of the size of about 5 mm. Hence, instead we should choose the smaller box of 4x4x3 mm cut by a ball of diameter 5 mm. This was the exact form of manna! One grain of manna is about the volume of one grain of wheat or barley. There are $144x10^4$ grains in the socket or 480 grains per shekel. Thus, the amount of silver that complements the grain to the box is 1/480 part of the shekel. (Remark 61: there is an interesting relation between the Troy ounce and the shekel of Torah. The Troy ounce is exactly 9/5 of the shekel. Since the Troy ounce consists of 480 grains, a Troy grain is 9/5 of a grain of manna). Half of it is the minimal monetary unit called pruta. This is the source of the tradition that pruta is an amount of silver that weights half of a barley grain (see Rambam, Mishne Torah, Shekalim, Ch.1, 3) (Remark 62: we will see in §26 that actually it was half of the wheat grain). Thus, there are 960 pruta in the shekel. This unit has a very simple geometric definition. It equals the amount of silver in four corners of the grain. To produce this amount, one should put together four grains to form a box 8x8x3 mm and hammer silver into the cavity in the middle of its upper face. (Remark 63: The weight of this box is 0.144 gr exactly as the Italian isar that was 1/96 of the shekel of the second Temple of 13.824 gr). According to Torah, shekel is divided into 20 gera. This unit thus corresponds to 24 grains that can be combined into 3x4x2 grains forming a box of 12x12x8 mm. Twenty such boxes form a box of 6x8x10 grains of 24x24x40 mm that corresponds to a shekel. The volume of this box is exactly the volume (and approximately the form) of an olive=1/2.5 of egg. This is the minimal unit of food. The relation 2.5 olives=1 egg solves the famous controversy between Rambam and Tosfot (according to Rambam olive is a little less than 1/3 of an egg and according to Tosfot it is 1/2 of an egg). The daily portion of manna was omer of 3/10 seah or 43.2 eggs. It is equal to an integer number of 108 olives and its weight is 108 shekels. This amount of manna could be arranged into a box of 72mm x72 mm and of the height of the socket- 48 cm The secular mane of 25 shekels can be arranged as the box of 30x40x10 grains of the dimension 12x12x4 cm. The mane of Sanctuary of 50 shekels was forming the box of 12x12x8 cm. These are the most natural dimensions for the socket: 12 cm is the thickness of its walls while 8 cm is the palm. The height of the socket of 6 palms is divided into six layers, in each layer there are 10 boxes of 12x12x8 cm. The Roman-Greek measures were hinted by the box of 12x12x12 cm. Its weight was 1296 gr, that is four Roman pounds or three Attic mina. Finally, the talent of Sanctuary of 3000 shekel was the total amount of silver in the socket. Thus the whole monetary system of Torah was defined by the manna.

The price of a minimal unit of bread is also linked to pruta. The standard price of wheat flour was four seah per shekel (e.g. Mishna Peah 8, 7). Since seah is equal to 144 eggs and egg

according to our suggestion is 2.5 olives, four seah are 1440 olives. Since there are 960 pruta in shekel, one pruta is the price of 1.5 olive of wheat. The price of bread consists of the price of flour, the price of wood (1/6 of the flour) and the labor of the baker (1/3 of the flour) (see Yiruvin 82b). If one gathers wood by himself and pays only for the labor of the baker, the price of an olive of bread is exactly one pruta. A poor man is given charity of pondion (1/48 of shekel) per day (Peah 8, 7). With this amount he can buy 12 eggs of wheat. If he pays for the bread 1/3, then he is left with eight eggs of bread. This is the higher of two standards of the daily amount of food as mentioned in §19. Thus our definition of pruta fits the higher standard. There is however another opinion. The Talmud in Kidushin 12a brings two opinions regarding the weight of pruta. According to the Rabies pruta is 1/768 of a shekel. According to Raban Shimon Ben Gamliel it is 4/3 times bigger. Both talk about the shekel of the second Temple. The first opinion agrees with our definition. Indeed, if the shekel of Torah of 17.28 gr was 960 pruta then the shekel of the first Temple of 11.52 gr was 640 pruta and the shekel of the second Temple of 13.824 gr was 768 pruta. But what is the meaning of the pruta of Raban Shimon Ben Gamliel? If a poor man pays also for the wood, than for one pondion he will get 6 eggs of bread- the minimal standard. Hence, in order to buy an olive of bread he will need not a pruta but 4/3 times more. This is the basis of the second opinion. It does not mean that Raban Shimon Ben Gamliel would reject our measures of manna. He could say that pruta was not half the weight of wheat of density 3/4 filling the volume 4x4x3 mm but of water of this volume. Notice that the basic unit gera of Torah of 48 pruta and gera of the first and second Temple of 32 pruta, are also multiples of the larger pruta.

What is the importance of pruta? According to the Jewish law a woman can be sanctified with a pruta. Why? Because she can buy with it an olive of bread and fulfill the commandment of eating matzo on Passover!

Manna could be used also in order to measure length. The width of five grains of manna is 20 mm- the thumb, four thumbs is the palm and six palms is the cubit.

The manna was covering the ground by a thin layer, "as the hoar frost" (Exodus 16:14). According to our calculation, the layer was 3 mm thick, as the thickness of a single grain. The portion of manna was omer of 3/10 of seah, or 2488.32 cm^3. Hence it covered the area of 8294.4 cm^2. This area is equal 48 cm times 3x57.6 cm. Recall that 48 cm is the cubit of Torah according to Rabbi Haim Naeh and 57.6 cm is the cubit according to Hazon Yish. Three bigger cubits fit the height of a man including the head, the smaller cubit fits the width of the shoulders. Thus the portion of manna covered the same area as man's body. If a person wanted to acquire his portion of manna, he was laying on it. If he wanted acquire manna also for his family (a standard family of four) he was lying on the ground and spreading his hands and legs apart. This way he was covering an area four cubits wide. This equality reconfirms our claim about the thickness of manna.

25. This paragraph is a part of our commentary on the design of the Third Temple in Ezekiel 40-43. The plan of the Temple and its courts is exhibited in the maps 1, 2, 3, 4, 5, 6 (in Hebrew). The crucial point in understanding this design is the connection between the Temple and the Tabernacle. In particular, the inner court of 100 by 100 cubits in front of the Temple

and the adjacent three outer courts of 100 by 100 cubits (see [3]) are analogous to the eastern half of the inner court of 50 by 50 cubits in front of the Tabernacle. The inner court of the Tabernacle was surrounded by columns. These columns were a cubit wide and thick. They were standing on brass sockets similarly to the boards of the Tabernacle that stood on silver sockets. The outer dimension of the sockets was 1x1x1 cubits. The counterparts of the columns of the court of the Tabernacle are the posts in Ezekiel 40:9-10 and 40:14. They were cylinders two cubits wide and sixty cubits tall. The counterpart of the sockets were little chambers תאים (taim) mentioned there is verses 7 and 10. Their size was 6 by 6 cubits (verse 7) and their height was also 6 cubits (verse 5). They had windows (verse 16). These windows are called "atumim"-sealed, as the windows of the Temple (Kings I 6:4). In both cases the windows were in the roof and were wider outside than inside. The purpose of the windows in these chambers was to accommodate the posts. Hence their exterior diameter was 2 cubits as of the posts. The lower six cubits of the posts and the windows were of conic form narrowing towards the bottom, so that the chambers would hold the posts tight. What material the chambers were made of? It is written in Isaiah 60:17 "For brass I will bring gold etc". May be the brass sockets will be replaced by golden ones? Let us calculate the weight of these golden chambers. Recall that the cubit of Ezekiel is 51 cm. In case of cylindrical hole of diameter 2, the volume of the chamber is $6^3-6\pi$ cubic cubits or 26152.2 dec^3. With specific gravity of gold 19.3 and the shekel of 17.28 gr we obtain 9736 talents of Sanctuary. The amount is apparently meant to be 10,000 talents. If the hole is conical with diameter 2/3x2 cubits at the bottom, then we obtain 10012.27 talents (Remark 64: In the Talmud two thirds are always called "two arms". Thus, the ration 2/3 may hint to the two arms of the boards of the Tabernacle covered by the silver sockets). The amount of 10,000 talents of silver is what Haman paid to exterminate Jewish people. This is indeed the value of 600,000 men between age 20 and 60 since the value of a man of such age is 50 shekels (Leviticus 27:3). The above close fit gives an additional support to our claim regarding the weight of the shekel.

There are ten chambers on each side of each court, a total of 120. The combined amount of gold is 622.08 thousand tons. The annual worldwide production of gold is about 50 million troy ounces. The above amount is exactly 40 times bigger- the production of 40 years. (Remark 65: The remaining 12.27 times 120 talents are 12 times the weight of the golden calf. Indeed, it is written in Shemot Rabba 42 that the golden calf weighed 125 kantar (as the numerical value of מסכה) or 120 kantar (as the numerical value of מסך). Kantar is the Roman name of talent (see §18). The average of these two opinions is 122.5 talents (we disregard 0.02 talent). May be 120 times 10,000 talents will be brought by the nations of the world since "Erev Rav"- the multitude of nations who followed the Children Of Israel in the desert, initiated the idea of the golden calf. Because of it, Children of Israel spent 40 years in the desert. The twelve tribes will bring the weight of the golden calf each, since they actually sinned. The love of gold will be then transformed into the love of G-d)

As for the posts, our conjecture is that they will be made (by the Heaven) from manna. Their height of 60 cubits is the same as the height of manna (Yoma 66a) (Remark 66: Our conjecture is that the manna was initially piling up to this height in the court of the Tabernacle, forming two boxes of dimensions 15x30x60 cubits. The width 15 cubits is the space to the left and to the right of the gate of the court and 30 cubits is the space to the east of the altar. The

volume of the manna was equal exactly 2,400,000 portions of omer for 600,000 men and their families of four. From the court, the manna was dispersed around the camp. Similarly, the water of the well of Miriam originated at the court and was flowing from there around the camp).

26. In this paragraph we will discuss the relation between pruta and the grain of barley or wheat. According to our calculation, the weight of a grain of manna was 0.036 gr and pruta was half of it - 0.018gr. The weight of manna fits the average weight of a grain of wheat of the land of Israel. In the book "Growing of crops" by Dr. Kostrinsky, 1966 (vol.I, pp. 136-139) (the book is in Hebrew) are given weights of thousand grains of three main kinds of wheat of the land of Israel (Remark 67: The Israeli wheat is very homogeneous and has distinct features). The first kind is var. horano-leucurum. It is almost round (similarly to manna) and weighs 30-35 gr (Remark 68: the actual weight depends on many conditions and varies from field to field and from year to year). The second kind var. leucurum, weighs 35-40 gr. The third kind var. melanopus, weighs 32-38 gr. The average weight is 35 gr. Thus the grains of wheat resemble manna in form, in weight and also in specific gravity of about 0.75. Kostrinsky also brings thousand grain weights of three selections from the local barley (vol. II, pp. 63-64). One is M.38-weight 45-48 gr, second F.48 -40-47 gr, third B.M.C -48-55 gr. The average weight is 47.17 gr. The length of barley grains exceeds their width by factor 3-4. The specific gravity of barley is about 0.61. Thus it does not resemble manna in any of the three features. Notice that the average weight of barley grain as almost exactly 4/3 of the wheat grain. Hence the pruta of Raban Shimon Ben Gamliel (see §24) is equal to half the barley grain which in turn is equal to the volume of manna of water.

It is interesting to compare the relation between grain and Torah weights with the ones in the Western and Arab world. The English system of weights is based of the Troy grain of 0.0648 gr. English pennyweight was equal to 24 Troy grains. Twenty penny made up the Troy ounce. This system resembles the division of the shekel of the Torah into twenty gera where each gera equals (according to our claim) to the weight of 24 grains of manna. Yet the Troy grain is much heavier than manna. There is an exact and simple relation between them: Troy grain is 9/5 of manna. The same penny was defined by Henry VII "to be of the weight of thirty-two grains of wheat that grew in the midst of the ear of wheat according to the old laws of this land". William Ridgeway in Metrological notes, (The Journal of Hellenic Studies, Vol. 10. (1889), pp. 90-97) identifies the Troy grain with grain of barley and claims that it is 4/3 of the grain of wheat. The ratio is the same we observed in the Israeli barley and wheat, yet the absolute weight of English grain is 27/20 larger than their Israeli counterpart. Perhaps, the reason is that English grain was taken from the midst of the ear while the weight of Israeli grain was based on the average. Notice that Israeli "theoretical" barley grain of 0.048 gr is almost exactly the Troy wheat grain of 0.486 gr. Their ratio is 80/81. The solidus of Constantine was divided into 24 siliquae. Ridgeway quotes a source in Hultsch (Metrl. Script. ii. 128) that siliqua equals three grains of barley. Thus solidus of Constantine was 72 barley grains or 96 wheat grains. Since the solidus was about 4.444 gr, the corresponding barley grain was about 0.0617 gr.

Now we will discuss Arab weights. The site quotes Ibn Khaldun saying "Know that there is consensus [ijma] since the beginning of Islam and the age of the Companions and the Followers that the dirham of the shari'ah is that of which ten weigh seven mithqals weight of the dinar of gold... The weight of a mithqal of gold is seventy-two grains of barley, so that the dirham which is seven-tenths of it is fifty and two-fifths grains. All these measurements are firmly established by consensus". Since mithqal is close to the solidus, the corresponding barley grains are the heavy ones. Notice that besides the coins of dinar and dirham there were weights with the same name. Walther Hinz in "Islamische Masse and Gewichte", brings the dirham weight of 3.125 gr and the corresponding mithqal of 4.464 gr. This weight is almost identical with the solidus of Constantine. Thus, the corresponding barley grain is close to the one of Constantine. If the wheat is 3/4 of barley as in English and Israeli grain, then mitqal weighs 96 grains of wheat. Hintz (p.4) also brings the weight of Egyptian mitqal of 4.68 gr (more precisely 4.6875 gr) which is 3/2 of the dirham of 3.125 gr. (Remark 69: This mitqal according to Hinz is exactly 1/72 of the Roman-Egyptian pound, which is 25/24 of the Roman pound of 324 gr (see §9)). We mention this weight since Rambam in his commentary of Mishna Bechorot 8, 7 writes "I found that the weight of Egyptian drachmon is sixty one grains of barley". If this drachmon is the standard dirham of 2.97 gr then the dinar of 96 barley grains is 4.67 gr, almost as the said Egyptian mitqal. The corresponding barley grain of Rambam is then **0.0487** gr, about the same as the English wheat grain of 0.0486 gr. If this is the opinion of Rambam, then the length of cubit which follows from his estimate of quarter of lug (see §14) is 47.18 cm, close to the measure of Rav Chaim Naeh. Another possibility is that Rambam identifies the Arab golden dinar with the dinar of Mishna and Talmud as do Gaonim and many Rishonim (see the quotations in the book of Rav Benish Ch. 30, foot notes 25-29) while the Egyptian dirham of his time was lighter than the standard (light dirham) of 2/3 of the mitqal. Since he equates the dinar of Mishna and Talmud with the weight of 96 barley grains and since the Arabic golden dinar is about 4.25 gr, the corresponding barley grain is about **0.0043** gr. Both weights are in the range of average barley grains of the Land of Israel. Yet, the first weight is very close to 0.048 gr of our "theoretical" barley that is 4/3 of the "theoretical" wheat and manna of 0.036 gr. Notice that the identification of mitqal with the weight of 96 barley grains instead of 96 wheat grains is not an "invention" of Gaonim. Kaftor Vaperach, ch. 16 writes that dirham is 16 qirt, and each qirt is 4 barley grains. He says that this fact is written in the book of Ibn Sina, and this is the custom among the merchants and the folk. The corresponding mitqal of 24 qirt is thus 96 barley grains. In Arabic, this small unit is called "habba" which means grain, without specifying of what kind.

Since Gaonim, Rif and Rambam equate the weight of the shekel of the Mishna and Talmud with the weight of 384 barley grains, and since according to the Rabies in Kidushin 12a pruta is 1/768 of this shekel, this pruta weighs as a half barley grain. There is however an extraordinary opinion of Rav Hai Gaon that pruta of Torah is a quarter of habba. We suggest the following explanation of this opinion. Hinz writes in his book (p.4) that according to Ibn Mu'ad the mitqal of gold in Iraq was divided into 20 qirt and each qirt into 3 habba. With mitqal equal 4.25 gr we obtain habba of 0.0708 gr and pruta **0.0177** gr, almost exactly the pruta of 0.018 gr according to our claim. Apparently, the same opinion is expressed by Shaarei

Shevuot and Sefer Hahinuch (see the book of Rav Benish, Ch. 22, note 46) which say that the shekel of Shlomo was 160 barley grains.

There is another confusing opinion of Hari Migash (הר"י מיגאש) (1077-1141) who was a student of Rif and a teacher of Rambam. He writes on Shevuot 39b concerning the issue of an oath by one who admits part of the claim, that "two money (of the claim) are dinars of the land – one eights of the Tyrian dinar. And Tyrian dinar is seventy eight habba, and half habba is pruta". Rav Benish (Ch. 21 note 14 and note 62) writes that this opinion is shared by Bahag, Baal Haitur, teachers of Rambam and Rav Hai Gaon. Our interpretation of this statement is as follows. Hari Migash identifies Tyrian dinar with the mitqal of weight of 78 habba, while the mitqal of coin according to Ibn Khaldun is identified with 72 barley grains. Since pruta is assumed to be half barley grain, the habba of Hari Migash is 1/72 of the golden dinar of 4.25 gr. Hence "two money" are 78/4=19.5 habba or 4.25/72x19.5=**1.151** gr. On the other hand, the accepted opinion is that two money are the two meah -1/12 of the sela of the second Temple. According to our calculation, it is 13.824/12=**1.152** gr, practically the same as the above. The mitqal of weight of Hari Migash is thus about 4.60 gr, close to the golden solidus. This fits the first opinion of Rash that Tyrian dinar is the golden of Constantine. Did Hari Migash and others carry an independent tradition of the weight of meah of the second Temple or it is Hashgacha Pratit (Personal Providence) that led them to the correct result?

27. **Conclusion**

It seems from all the arguments we collected that the shekel of the first Temple was 11.52 gr and the shekel of the second Temple was 6/5 heavier, or 13.824 gr. There was also a little heavier standard of 25/24 of this shekel, namely 14.4 gr. The pruta according to all opinions, but Raban Shimon ben Gamliel (Rashbag), was 1/640 of the shekel of the first Temple and 1/768 of the second Temple. Its weight was thus 0.018 gr that is about one half of the average wheat grain of the Land of Israel. According to Rashbag pruta was 4/3 heavier or 0.024 gr that is about one half of the average barley grain of the Land of Israel. Yet Rashbag did not dispute the Rabies (see Kidushin 12a) regarding the weight of the shekel. It seems that the Gaonim, Rif and Rambam "inherited" the opinion of Rashbag about the weight of pruta while assuming that it is 1/768 of the shekel of the second Temple, which is the opinion of Rabies. Thus, they obtained a heavy shekel that was about the weight of the Attic tetra drachma or four golden solidi of Constantine or four Arabic mitqal. Therefore, from the rational point of view this opinion is a mistake. Yet it is very hard to assume that all Torah authorities of Babylon as well as Spain made such a huge mistake. We saw in §12 that Rashi too held that the shekel of the second Temple was equal to four golden solidi of Constantine and it is possible that his second opinion (that it is half of Cologne ounce) is a result of a decrease of the weight of denier. The weight of shekel was continuously maintained by the Jewish community since for every firstborn son, the father had to pay 5 shekels of silver to kohen. This is a large amount and people were not eager to overpay. Our claim that the shekel of Torah was initially 17.32 gr and was subsequently reduced by 1/3 and again increased, may explain this puzzle. The tradition of the heavy shekel was constantly kept by the Hebrew nation. Yet the contribution to the Temple was always half of the current shekel. The price of grain- four seah of wheat per shekel was also fixed in the terms of the current shekel.

Apparently, the five shekels of firstborn were also linked to the current shekel since the prices and wages were linked to it. Yet, many Jews paid to kohanim five shekels of the Torah - the amount for which Joseph was sold. During the Greek rule in Israel, Attic tetra drachma became a common standard in Israel. Thus, even more Jews paid for firstborn five Attic tetra drachmas. In Babylon, after the conquest of Alexander the Great, the currency became of the Attic standard. Later, in the Sassanian empire, the silver dinar was about the Attic standard. In Byzantine empire, the basic coin solidus was also close to the Attic drachma. After Islamic conquest, the golden dinar was also about the Attic drachma. Hence, the Jews of Babylon, of Egypt and also of Arabic Spain adopted the heavy standard of the shekel. The tradition of pruta of half wheat grain became confused with the half barley grain since some followed the opinion of Rashbag. In addition, the weight of the heavy wheat grain from the middle of the ear was about the same as the average barley grain. When compared with the mitqal or solidus, this barley grain fit the theoretical ratio 1:764 of the pruta to the shekel. All these factors reinforced the opinion of Gaonim and Rambam. On the other hand, in Rome, the silver dinarius was lighter but very close to the Tyrian dinar. Hence, the Jews there maintained the standard of the second Temple. Yet they were influenced by Jews from Spain and from Babylon. This resulted in two conflicting traditions. Since the local standards were constantly changing, it was also easy to loose the track of the correct shekel. Thus by the ways of Providence the weight of the shekel was increased twice and returned to its original value.